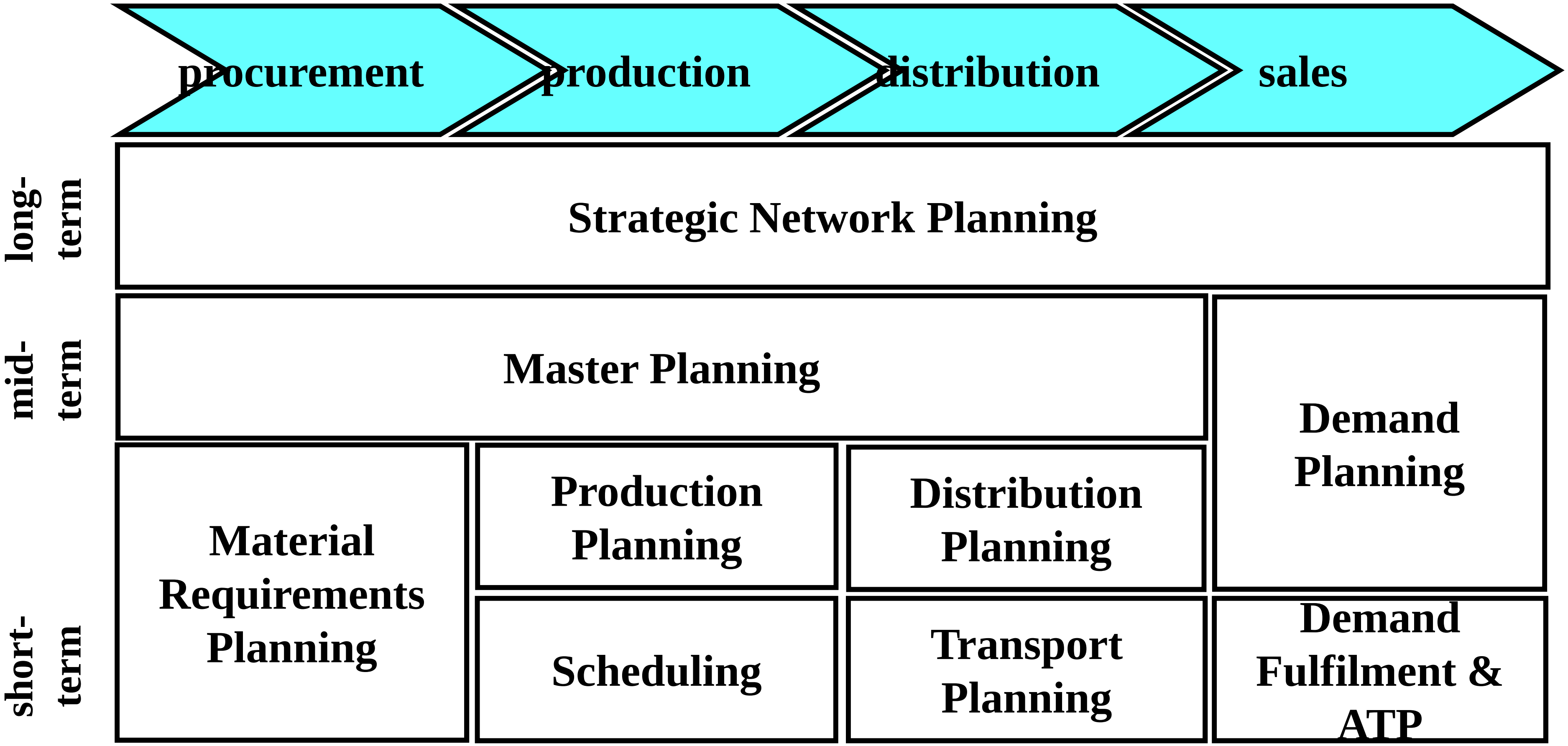
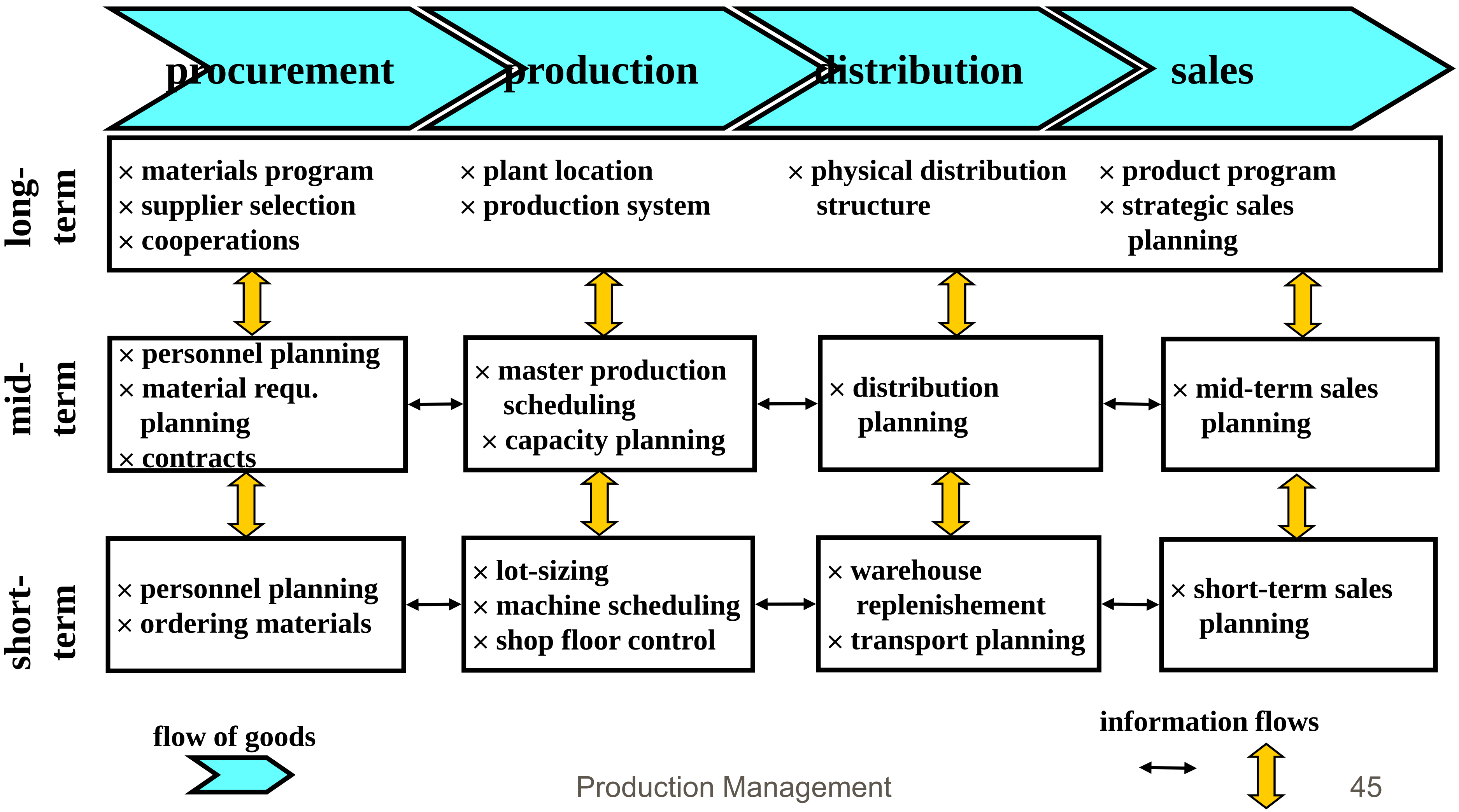
Chapter 5

Aggregate Planning

Supply Chain Planning Matrix



Supply Chain Planning Matrix



Aggregate Planning

- **Example:**
  - one product (plastic case)
  - two injection molding machines, 550 parts/hour
  - one worker, 55 parts/hour
  - steady sales 80.000 cases/month
  - 4 weeks/month, 5 days/week, 8h/day
  - how many workers?

- **in real life constant demand is rare**
  - change demand
  - produce a constant rate anyway
  - vary production

Aggregate Planning

- **Influencing demand**
  - do not satisfy demand
  - shift demand from peak periods to nonpeak periods
  - produce several products with peak demand in different period

- **Planning Production**
  - Production plan: how much and when to make each product
  - rolling planning horizon
  - long range plan
  - intermediate-range plan
    - units of measurements are aggregates
    - product family
    - plant department
    - changes in workforce, additional machines, subcontracting, overtime,…
  - Short-term plan

Aggregate Planning

- **Aspects of Aggregate Planning**
  - Capacity: how much a production system can make
  - Aggregate Units: products, workers,…
  - Costs
    - production costs (economic costs!)
    - inventory costs(holding and shortage)
    - capacity change costs

Aggregate Planning

- **Spreadsheet Methods**
- **Zero Inventory Plan**
  - Precision Transfer, Inc. Produces more than 300 different precision gears (the aggregation unit is a gear!).
  - Last year (=260 working days) Precision made 41.383 gears of various kinds with an average of 40 workers.
  - 41.383 gears per year
  - *40 x 260 worker-days/year = 3,98 -> 4 gears/ worker-day*

  - Aggregate demand forecast for precision gear:

Month	January	February	March	April	May	June	Total
Demand	2760	3320	3970	3540	3180	2900	19.670

Aggregate Planning

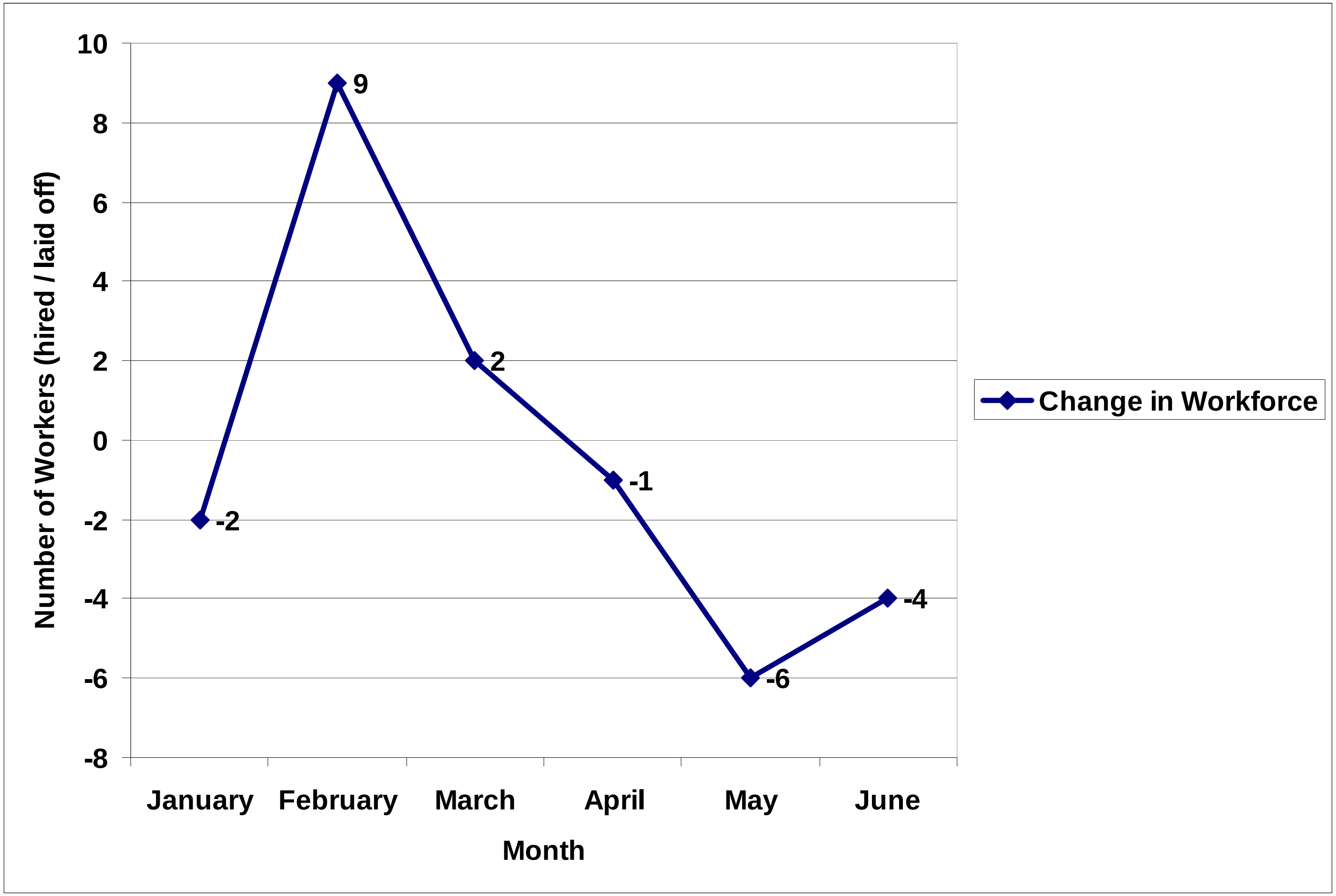
- holding costs: $5 per gear per month
- backlog costs: $15 per gear per month
- hiring costs: $450 per worker
- lay-off costs: $600 per worker
- wages: $15 per hour (all workers are paid for 8 hours per day)
- there are currently 35 workers at Precision
- currently no inventory

- Production plan?

Aggregate Planning

- **Zero Inventory Plan**
  - produce exactly amount needed per period
  - adapt workforce

Aggregate Planning



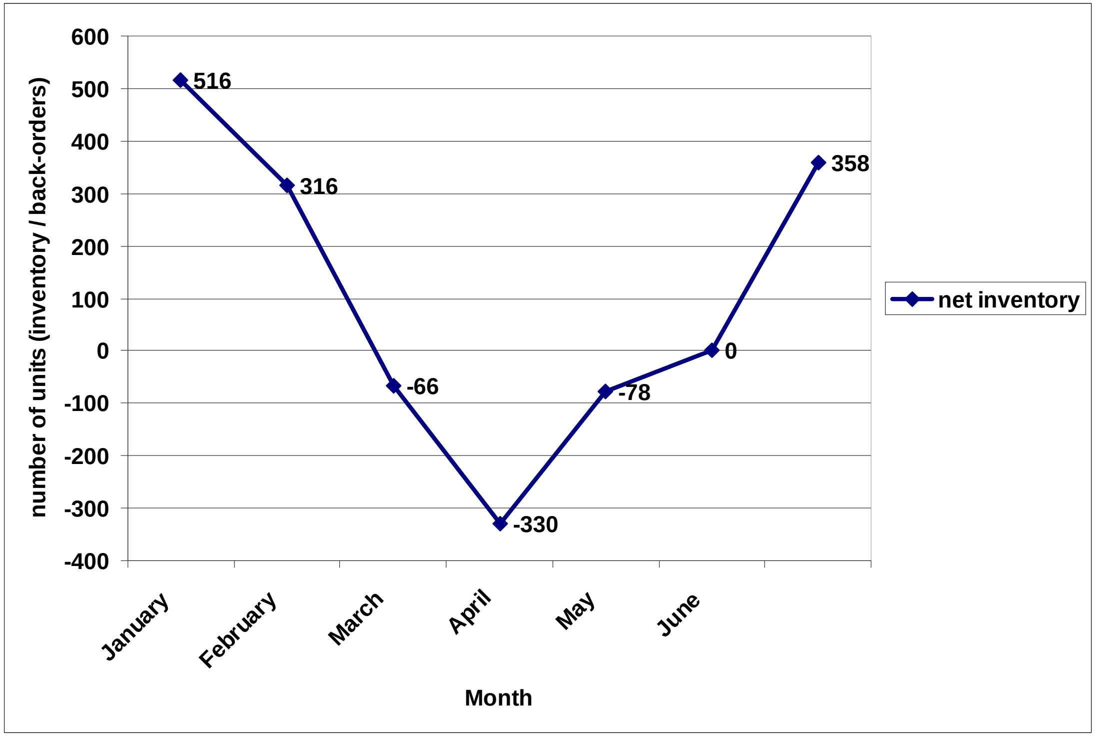
Aggregate Planning

- **Level Work Force Plan**
  - backorders allowed
  - constant numbers of workers
  - demand over the planning horizon
  - gears a worker can produce over the horizon

  - 19670/(4x129)=38,12 -> 39 workers are always needed

Aggregate Planning

- Inventory: January: 3276 - 2760 = 516
- February: 516 + 3120 – 3320
- March: 316 + 3588 – 3670 = -66! -Backorders: 66 x $15 = $990



Aggregate Planning

- **no backorders are allowed**
  - *workers= cumulative demand/(cumulative days x units/workers/day)*

  - January: 2760/(21 x 4) = 32,86 -> 33 workers
  - February: (2760+3320)/[(21+20) x 4] = 37,07 -> 38 workers.
  - March: 10.050/(64 x 4) =>40 workers
  - April: 13.590/(85 x 4) => 40 workers
  - May: 16.770/(107 x 4) => 40 workers
  - June: 19670/(129 x 4) => 39 workers

Aggregate Planning

- **Example Mixed Plan**
  - The number of workers used is an educated guess based on the zero inventory and level work force plans!

Spreadsheet Methods Summary

	Zero-Inv.	Level/BO	Level/No BO	Mixed
Hiring cost	4950	1800	2250	3150
Lay-off cost	7800	0	0	4200
Labor cost	59856	603720	619200	593520
Holding cost	0	4160	6350	3890
BO cost	0	7110	0	990
Total cost	611310	616790	627800	605180
Workers	33	39	40	35

Aggregate Planning

- **Linear Programming Approaches to Aggregate Planning**

Parameters:

T... Planning horizon length

t ... Index of periods, t=1,2,..., T

D_t ...forcasted number of units demanded in period t

n_t ...number of units that can be made by one worker in period t

C_t^P ...cost to produce one unit in period t

C_t^W ...cost of one worker in period t

Aggregate Planning

C_t^H …cost to hire one worker in period t

C_t^L …cost to lay off one worker in period t

C_t^I …cost to hold one unit in inventory in period t

C_t^B …cost to backorder one unit in period t

Aggregate Planning

Decision Variables:

P_t …number of units produced in period t

W_t …number of workers available in period t

H_t …number of workers hired in period t

L_t …number of workers laid off in period t

I_t …number of units held in inventory in period t

B_t …number of units backordered in period t

Aggregate Planning

Constraints: work, Capacity, force, material

$$\mathrm{P_t} \le n_t W_t \qquad t = 1, 2, \ldots, T$$

$$\mathrm{W_t} = W_{t-1} + H_t - L_t \qquad t = 1, 2, \ldots, T$$

net inventory this period = net inventory last period + production this period - demand this period

$$\mathrm{I_t} - B_t = \mathrm{I_{t-1}} - B_{t-1} + P_t - D_t$$

Costs

$$\sum_{t=1}^{T} (C_t^P P_t + C_t^W W_t + C_t^H H_t + C_t^L L_t + C_t^I I_t + C_t^B B_t)$$

Aggregate Planning

- **Example: Precision Transfer**
  - Planning horizon: 6 months T= 6
  - Costs do not vary over time $C_t^P = 0$
  - d_t : days in month t
  - $C_t^W = \$120 d_t$
  - $C_t^H = \$450$
  - $C_t^L = \$600$
  - $C_t^I = \$5$

  - We assume that no backorders are allowed!
  - no production costs and no backorder costs are included!
  - Demand

January	February	March	April	May	June	Total
2760	3320	3970	3540	3180	2900	19.670

Linear Program Model for Precision Transfer

Minimize

$$2520W_1 + 2400W_2 + 2760W_3 + 2520W_4 + 2640W_5 + 2640W_6$$
$$+ 450(H_1 + H_2 + H_3 + H_4 + H_5 + H_6)$$
$$+ 600(L_1 + L_2 + L_3 + L_4 + L_5 + L_6)$$
$$+ 5(I_1 + I_2 + I_3 + I_4 + I_5 + I_6)$$

subject to
(Production-capacity constraints)

$$P_1 \leq 84W_1, \quad P_2 \leq 80W_2, \quad P_3 \leq 92W_3, \quad P_4 \leq 84W_4, \quad P_5 \leq 88W_5, \quad P_6 \leq 88W_6,$$

(Work-force constraints)

$$W_1 = 35 + H_1 - L_1, \quad W_2 = W_1 + H_2 - L_2, \quad W_3 = W_2 + H_3 - L_3,$$
$$W_4 = W_3 + H_4 - L_4, \quad W_5 = W_4 + H_5 - L_5, \quad W_6 = W_5 + H_6 - L_6,$$

(Inventory-balance constraints)

$$I_1 = P_1 - 2760, \quad I_2 = I_1 + P_2 - 3320, \quad I_3 = I_2 + P_3 - 3970,$$
$$I_4 = I_3 + P_4 - 3540, \quad I_5 = I_4 + P_5 - 3180, \quad I_6 = I_5 + P_6 - 2900\,(= 0),$$

(Non-negativity constraints)

$$P_1, P_2, P_3, P_4, P_5, P_6, W_1, W_2, W_3, W_4, W_5, W_6, H_1, H_2, H_3, H_4, H_5, H_6,$$
$$L_1, L_2, L_3, L_4, L_5, L_6, I_1, I_2, I_3, I_4, I_5, I_6 \geq 0$$

Aggregate Planning

- **LP solution (total cost = $600 191,60)**

	Production	Inventory	Hired	Laid off	Workers
January	2940,00	180,00	0,00	0,00	35,00
February	3232,86	92,86	5,41	0,00	40,41
March	3877,14	0,00	1,73	0,00	42,14
April	3540,00	0,00	0,00	0,00	42,14
May	3180,00	0,00	0,00	6,01	36,14
June	2900,00	0,00	0,00	3,18	32,95

Aggregate Planning

- **Rounding LP solution**

	January	February	March	April	May	June	Total
Days	21	20	23	21	22	22	129
Units/Worker	84	80	92	84	88	88	516
Demand	2760	3320	3970	3540	3180	2900	19670
Workers	**35**	**41**	**42**	**42**	**36**	**33**	229
Capacity	2940	3280	3864	3528	3168	2904	19684
Capacity - Demand	180	-40	-106	-12	-12	4	14
Cumulative Difference	180	140	34	22	10	14	400
Produced	2930	3280	3864	3528	3168	2900	19670
Net inventory	170	130	24	12	0	0	336
Hired	0	6	1	0	0	0	7
Laid Off	0	0	0	0	6	3	9
Costs	89050	101750	116490	105900	98640	88920	**600750**

Aggregate Planning

- **Practical Issues**
  - 100.000 variables and 40.000 constraints
  - LP/MIP Solvers: CPLEX, XPRESS-MP, ...
- **Extensions**
  - Bounds

$$\mathrm{I}_t \le I_t^U$$

$$\mathrm{I}_t^L \le \mathrm{I}_t \le I_t^U$$

$$\mathrm{L}_t \le 0.05 W_t$$

Training

$$W_t = W_{t-1} + H_{t-1} - L_t$$

Aggregate Planning

- **Transportation Models**
  - supply points: periods, initial inventory
  - demand points: periods, excess demand, final inventory

$n_t W_t$ = capacity during period t

D_t = forecasted number of units demanded in period t

C_t^P = the cost to produce one unit in period t

C_t^I = the cost to hold one unit in inventory in period t

Aggregate Planning

t	1	2	3
capacity n_tW_t	350	300	350
demand	200	300	400
production costs	10	11	12
holding costs	2	2	2

initial inventory: 50
final inventory: 75

Aggregate Planning

	1	**2**	**3**	Ending inventory	Excess capacity	Available capacity
Beginning inventory	0 **50**	2	4	6	0	50
Period 1	10 **150**	12	14 **50**	16 **75**	0 **75**	350
Period 2	-	11 **300**	13	15	0	300
Period 3	-	-	12 **350**	14	0	350
Demand	200	300	400	75	75	1050

Aggregate Planning

- Extension:

t	1	2	3
capacity $n_t W_t$	350	350	300
demand	400	300	400
production costs	10	11	12
holding costs	2	2	2

  - **overtime**: overtime capacity is **90, 90** and **75** in period 1, 2 and 3;
  - overtime costs are **$16, $18** and **$ 20** for the three periods respectively;
  - **backorders**:units can be backordered at a cost of **$5** per unit-month; production in period 2 can be used to satisfy demand in period 1

Aggregate Planning

		1	2	3	Ending inventory	Excess capacity	Available capacity
Beginning inventory		0	2 **25**	4 **25**	6	0	50
Period 1	Regular time	10 **350**	12	14	16	0	350
	Overtime	16 **50**	18	20	22	0 **40**	90
Period 2	Regular time	16	11 **275**	13	15 **75**	0	350
	Overtime	23	18	20	22	0 **90**	90
Period 3	Regular time	22	17	12 **300**	14	0	300
	Overtime	30	25	20 **75**	22	0	75
Demand		400	300	400	75	130	1305

Aggregate Planning

- **Disaggregating Plans**
  - aggregate units are not actually produced, so the plan should consider individual products
  - disaggregation
  - master production schedule

- **Questions:**
  - In which order should individual products be produced?
    - e.g.: shortest run-out time $R_i = I_i / D_i$
  - How much of each product should be produced?
    - e.g.: balance run-out time

Aggregate Planning

- **Advanced Production Planning Models**
  - Multiple Products
  - same notation as before
  - add subscript i for product i

  - Objective function

$$\min \sum_{t=1}^{T} \left(C_t^W W_t + C_t^H H_t + C_t^L L_t + \sum_{i=1}^{N} C_{it}^P P_{it} + C_{it}^I I_{it} \right)$$

Aggregate Planning

subject to

$$\sum_{i=1}^{N}\left(\frac{1}{n_{it}}\right)P_{it} \le W_t \qquad t = 1, 2, \ldots, T$$

$$W_t = W_{t-1} + H_t - L_t \qquad t=1,2,\ldots,T$$

$$I_{it} = I_{it-1} + P_{it} - D_{it} \qquad t=1,2,\ldots,T;\ i=1,2,\ldots,N$$

$$P_{it}, W_t, H_t, L_t, I_{it} \ge 0 \qquad t=1,2,\ldots,T;\ i=1,2,\ldots,N$$

Aggregate Planning

- **Computational Effort:**
  - 10 products, 12 periods: 276 variables, 144 constraints
  - 100 products, 12 periods: 2436 variables, 1224 constaints

Aggregate Planning

- Example: Carolina Hardwood Product Mix
  - Carolina Hardwood produces 3 types of dining tables;
  - There are currently 50 workers employed who can be hired and laid off at any time;
  - Initial inventory is 100 units for table1, 120 units for table 2 and 80 units for table 3;

t	1	2	3	4
costs of hiring	420	410	420	405
costs of lay off	800	790	790	800
costs per worker	600	620	620	610

Aggregate Planning

- The number of units that can be made by one worker per period:

t	Table 1	Table 2	Table 3
1	200	300	260
2	220	310	255
3	210	300	250
4	200	290	265

- Forecasted demand, unit cost and holding cost per unit are:

	Demand			Unit costs			Holding costs		
t	Table 1	Table 2	Table 3	Table 1	Table 2	Table 3	Table 1	Table 2	Table 3
1	3500	5400	4500	120	150	200	10	12	12
2	3100	5000	4200	125	150	210	9	11	12
3	3000	5100	4100	120	145	205	10	12	11

Aggregate Planning

Minimize

$$600W_1 + 620W_2 + 620W_3 + 610W_4 + 420H_1 + 410H_2 + 420H_3 + 405H_4 + 800L_1 + 790L_2 + 790L_3 + 800L_4$$
$$+ 120P_{11} + 150P_{21} + 200P_{31} + 125P_{12} + 150P_{22} + 210P_{32} + 120P_{13} + 145P_{23} + 205P_{33} + 125P_{14} + 148P_{24} + 205P_{34}$$
$$+ 10I_{11} + 12I_{21} + 12I_{31} + 9I_{12} + 11I_{22} + 12I_{32} + 10I_{13} + 12I_{23} + 11I_{33} + 10I_{14} + 11I_{24} + 11I_{34}$$

subject to

$$\frac{P_{11}}{200} + \frac{P_{21}}{300} + \frac{P_{31}}{260} \le W_1, \quad \frac{P_{12}}{220} + \frac{P_{22}}{310} + \frac{P_{32}}{255} \le W_2,$$
$$\frac{P_{13}}{210} + \frac{P_{23}}{300} + \frac{P_{33}}{250} \le W_3, \quad \frac{P_{14}}{200} + \frac{P_{24}}{290} + \frac{P_{34}}{265} \le W_4,$$

$$W_1 = 50 + H_1 - L_1, \quad W_2 = W_1 + H_2 - L_2, \quad W_3 = W_2 + H_3 - L_3, \quad W_4 = W_3 + H_4 - L_4,$$

$$I_{11} = 100 + P_{11} - 3500, \quad I_{21} = 120 + P_{21} - 5400, \quad I_{31} = 80 + P_{31} - 4500,$$
$$I_{12} = I_{11} + P_{12} - 3100, \quad I_{22} = I_{21} + P_{22} - 5000, \quad I_{32} = I_{31} + P_{32} - 4200,$$
$$I_{13} = I_{12} + P_{13} - 3000, \quad I_{23} = I_{22} + P_{23} - 5100, \quad I_{33} = I_{32} + P_{33} - 4100,$$
$$I_{14} = I_{13} + P_{14} - 3400, \quad I_{24} = I_{23} + P_{24} - 5500, \quad I_{34} = I_{33} + P_{34} - 4600,$$

$$P_{it}, I_{it}, W_t, H_t, L_t, I_{it} \ge 0$$

Aggregate Planning

- **Multiple Products and Processes**

T = horizon length, in periods
N = number of products
K = number of resource types
t = index of periods, $t = 1, 2, \ldots, T$
i = index of products, $i = 1, 2, \ldots, N$
k = index of resource types, $k = 1, 2, \ldots, K$
D_{it} = forecasted number of units demanded for product i in period t
m_i = number of different processes available to make product i
A_{kt} = amount of resource k available in period t
a_{ijk} = amount of resource k required by one unit of product i if produced by process j
C^P_{ijt} = cost to produce one unit of product i using process j in period t
C^I_{it} = cost to hold one unit of product i in inventory for period t

The decision variables are

P_{ijt} = number of units of product i produced by process j in period t
I_{it} = number of units of product i held in inventory at the end of period t

Aggregate Planning

The linear programming formulation is

$$\text{Minimize} \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{m_i} (C_{ijt}^{P} P_{ijt} + C_{it}^{I} I_{it})$$

subject to

$$\sum_{i=1}^{N} \sum_{j=1}^{m_i} a_{ijk} P_{ijt} \leq A_{kt} \qquad t = 1, 2, \ldots, T; \;\; k = 1, 2, \ldots, K$$

$$I_{it} = I_{it-1} + \sum_{j=1}^{m_i} P_{ijt} - D_{it} \qquad t = 1, 2, \ldots, T; \;\; i = 1, 2, \ldots, N$$

$$P_{ijt}, I_{it} \geq 0 \qquad t = 1, 2, \ldots, T; \;\; i = 1, 2, \ldots, N; \;\; j = 1, 2, \ldots, m_i$$

Aggregate Planning

- Example: Cactus Cycles process plan
- CC produces 2 types of bicycles, street and road;
- Estimated demand and current inventory:

t	initial inventory	1	2	3
street b.	**100**	**1000**	**1050**	**1100**
road b.	**50**	**500**	**600**	**550**

- available capacity(hours) and holding costs per bike:

	Capacity(hours)		Holding	
t	Machine	Worker	Street	Road
1	8600	17000	5	6
2	8500	16600	6	7
3	8800	17200	5	7

Aggregate Planning

- process costs (process1, process2) and resource requirement per unit:

	Process1		Process2	
t	Street	Road	Street	Road
1	72	85	80	90
2	74	88	78	95
3	75	84	78	92
Machine hours required	5	8	4	6
Worker hours required	10	12	8	9

Aggregate Planning

Minimize

$$72P_{111} + 80P_{121} + 85P_{211} + 90P_{221}$$
$$+ 74P_{112} + 78P_{122} + 88P_{212} + 95P_{222}$$
$$+ 75P_{113} + 78P_{123} + 84P_{213} + 92P_{223}$$
$$+ 5I_{11} + 6I_{12} + 5I_{13} + 6I_{21} + 7I_{22} + 7I_{23}$$

subject to

$$5P_{111} + 4P_{121} + 8P_{211} + 6P_{221} \leq 8600, \quad 10P_{111} + 8P_{121} + 12P_{211} + 9P_{221} \leq 17000,$$
$$5P_{112} + 4P_{122} + 8P_{212} + 6P_{222} \leq 8500, \quad 10P_{112} + 8P_{122} + 12P_{212} + 9P_{222} \leq 16600,$$
$$5P_{113} + 4P_{123} + 8P_{213} + 6P_{223} \leq 8800, \quad 10P_{113} + 8P_{123} + 12P_{213} + 9P_{223} \leq 17200,$$

$$I_{11} = 100 + P_{111} + P_{121} - 1000, \quad I_{21} = 50 + P_{211} + P_{221} - 500,$$
$$I_{12} = I_{11} + P_{112} + P_{122} - 1050, \quad I_{22} = I_{21} + P_{212} + P_{222} - 600,$$
$$I_{13} = I_{12} + P_{113} + P_{123} - 1100, \quad I_{23} = I_{22} + P_{213} + P_{223} - 550,$$

$$P_{ijt}, I_{it} \geq 0 \qquad t = 1, 2, 3; \quad i = 1, 2; \quad j = 1, 2$$

Aggregate Planning

- **solution: Objective Function value = $368,756.25**

	Street Bicycle			Road Bicycle		
	Process			Process		
t	1	2	Inventory	1	2	Inventory
1	900	0	0	118,75	525	193,75
2	1050	0	0	406,25	0	0
3	0	1100	0	550	0	0

Aggregate Planning - Extensions

- **Hopp / Spearman, S. 522-540**
- **Notation:**

X_{it} ... amount of product i produced in period t

r_i … net profit from one unit of product i

S_{it} … amount of product i sold in period t

a_{ij} … time required on workstation j to produce one unit of product i

c_{jt} … capacity of workstation j in period t in units (consistent with a_{ij})

Aggregate Planning - Extensions

- **Backorders**

$$\max \sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} \mathrm{r_i} S_{it} - h_i I_{it}^{+} - \pi_i I_t^{-}$$

subject to

$$\underline{d}_{it} \le S_{it} \le \bar{d}_{it} \qquad \text{for all } i,t$$

$$\sum_{i=1}^{m} \mathrm{a_{ij}} X_{it} \le c_{jt} \qquad \text{for all } j,t$$

$$I_{it} = I_{it-1} + X_{it} - S_{it} \qquad \text{for all } i,t$$

$$I_{it} = I_{it}^{+} - I_{it}^{-} \qquad \text{for all } i,t$$

$$X_{it}, S_{it}, I_{it}^{+}, I_{it}^{-} \ge 0 \qquad \text{for all } i,t$$

Aggregate Planning - Extensions

- **Overtime**

l'_j = cost of one hour of overtime at workstation j

O_{jt} = overtime at workstation j in period t in hours

$$\max \sum_{t=1}^{\bar{t}} \left\{ \sum_{i=1}^{m} (r_i S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-}) - \sum_{j=1}^{n} l' O_{jt} \right\}$$

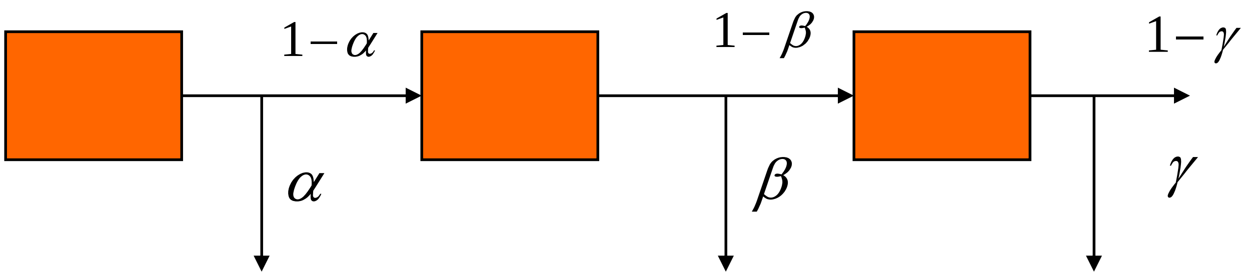
subject to

$$\sum_{i=1}^{m} a_{ij} X_{it} \le c_{jt} + O_{jt} \qquad \text{for all } i,t$$

$$X_{it}, S_{it}, I_{it}^{+}, I_{it}^{-}, O_{jt} \ge 0 \qquad \text{for all } i,t$$

Aggregate Planning - Extensions

- **Yield loss**



$\alpha, \beta, \gamma \ldots$ fraction of output that is lost

$y_{ij} \ldots$ cumulative yield from station j onward

(including station j) for product i

we must release $\frac{d}{y_{ij}}$ units of i into station j

Aggregate Planning - Extensions

- **Basic model + Yield loss extension (no backorders)**

$$\max \sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} (r_i S_{it} - h_i I_{it})$$

subject to

$$d_{it} \le S_{it} \le \bar{d}_{it} \qquad \text{for all } i,t$$

$$\sum_{i=1}^{m} \frac{a_{ij} X_{it}}{y_{ij}} \le c_{jt} \qquad \text{for all } j,t$$

$$I_{it} = I_{it-1} + X_{it} - S_{it} \qquad \text{for all } i,t$$

$$X_{it}, S_{it}, I_{it} \ge 0 \qquad \text{for all } i,t$$

Aggregate Planning - WorkforcePlanning

- **Single product, workforce resizing, overtime allocation**
- **Notation**

$\text{b} = \text{number of man-hours required to produce one unit of product}$

$\text{l} = \text{cost of regular time in dollars/man-hour}$

$l' = \text{cost of overtime in dollars/man-hour}$

$\text{e} = \text{cost to increase workforce by one man-hour per period}$

$\text{e}' = \text{cost to decrease workforce by one man-hour per period}$

$\text{W}_t = \text{workforce in period t in man-hours of regular time}$

$\text{H}_t = \text{increase in workforce from period t-1 to t in man-hours}$

$\text{F}_t = \text{decrease in workforce from period t-1 to t in man-hours}$

$\text{O}_t = \text{overtime in period t in hours}$

Aggregate Planning - WorkforcePlanning

- **LP formulation: maximize net profit, including labor, overtime, holding, and hiring/firing costs**
- **subject to constraints on sales, capacity,...**

$$\max \sum_{t=1}^{\bar{t}} \left\{ rS_t - h\, I_t - lW_t - l'O_t - eH_t - e'F_t \right\}$$

subject to

$$d_t \leq S_t \leq \bar{d}_t \qquad \text{for all } t$$

$$a_j X_t \leq c_{jt} \qquad \text{for all j, } t$$

$$I_t = I_{t-1} + X_t - S_t \qquad \text{for all } t$$

$$W_t = W_{t-1} + H_t - F_t \qquad \text{for all } t$$

$$bX_t \leq W_t + O_t \qquad \text{for all } t$$

$$X_{t,} S_{t,} I_t\,, O_{t,} W_{t,} F_{t,} H_{t,} \geq 0 \qquad \text{for all } t$$

AP-WP Example

- **Revenue: 1000$**
- **worker capacity: 168h/month**
- **initially 15 workers**
- **no initial inventory**
- **holding costs: 10$/unit/month**
- **regular labor costs: 35$/hour**
- **overtime: 150% of regular**
- **hiring costs: 2500$ (2500/168 ~ 15$ per man-hour)**
- **lay-off costs: 1500$ (1500/168 ~ 9$ per man-hour)**
- **no backordering**
- **demands over 12 months:**

200, 220, 230, 300, 400, 450, 320, 180, 170,170, 160, 180

- **demands must be met! (S=D)**

AP-WP Example(cont.)

- **Determine over a 12 month horizon:**
  - **Number of workers: W**
  - **Output: X**
  - **Overtime use: O**
  - **Inventory: I**
  - **(H, F are additional choice variables in the model)**

Aggregate Planning - WorkforcePlanning

Parameters:													
r	1000												
h	10												
l	35												
l'	52,5												
e	15												
e'	9												
b	12												
I_0	0												
W_0	2520												
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	total
	200	220	230	300	400	450	320	180	170	170	160	180	2980
Desicion Variables:													
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
Xt													
Wt													
Ht													
Ft													
It													
Ot													
Objective:													
Profit	2980000	$											

Aggregate Planning - WorkforcePlanning

Constraints:				
I1-I0-X1	0,00	=	-200	d_1
I2-I1-X2	0,00	=	-220	d_2
I3-I2-X3	0,00	=	-230	d_3
I4-I3-X4	0,00	=	-300	d_4
I5-I4-X5	0,00	=	-400	d_5
I6-I5-X6	0,00	=	-450	d_6
I7-I6-X7	0,00	=	-320	d_7
I8-I7-X8	0,00	=	-180	d_8
I9-I8-X9	0,00	=	-170	d_9
I10-I9-X10	0,00	=	-170	d_10
I11-I10-X11	0,00	=	-160	d_11
I12-I11-X12	0,00	=	-180	d_12
W1-W0-H1+F1	-2520,00	=	0	
W2-W1-H2+F2	0,00	=	0	
W3-W2-H3+F3	0,00	=	0	
W4-W3-H4+F4	0,00	=	0	
W5-W4-H5+F5	0,00	=	0	
W6-W5-H6+F6	0,00	=	0	
W7-W6-H7+F7	0,00	=	0	
W8-W7-H8+F8	0,00	=	0	
W9-W8-H9+F9	0,00	=	0	
W10-W9-H10+F10	0,00	=	0	
W11-W10-H11+F11	0,00	=	0	
W12-W11-H12+F12	0,00	=	0	

bX1-W1-O1	0,00	<=	0	
bX2-W2-O2	0,00	<=	0	
bX3-W3-O3	0,00	<=	0	
bX4-W4-O4	0,00	<=	0	
bX5-W5-O5	0,00	<=	0	
bX6-W6-O6	0,00	<=	0	
bX7-W7-O7	0,00	<=	0	
bX8-W8-O8	0,00	<=	0	
bX9-W9-O9	0,00	<=	0	
bX10-W10-O10	0,00	<=	0	
bX11-W11-O11	0,00	<=	0	
BX12-W12-O12	0,00	<=	0	

Aggregate Planning - WorkforcePlanning

Parameters:													
r	1000												
h	10												
l	35												
l'	52,5												
e	15												
e'	9												
b	12												
I_0	0												
W_0	2520												
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	total
	200	220	230	300	400	450	320	180	170	170	160	180	2980
Desicion Variables:													
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
Xt	302,86	302,86	302,86	302,86	302,86	302,86	302,86	180,00	170,00	170,00	170,00	170,00	2980,00
Wt	3634,29	3634,29	3634,29	3634,29	3634,29	3634,29	3634,29	2160,00	2040,00	2040,00	2040,00	2040,00	35760,00
Ht	1114,29	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1114,29
Ft	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1474,29	120,00	0,00	0,00	0,00	1594,29
It	102,86	185,71	258,57	261,43	164,29	17,14	0,00	0,00	0,00	0,00	10,00	0,00	1000,00
Ot	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Objective:													
Profit	1687337	$											

Aggregate Planning-Summary

The following scenarios have been discussed:

- **single product, single resource, single process**
  find: workforce, output, inventory (w. or w/o backorders)
- **multiple products, single resource, single process**
  find: workforce, all outputs, all inventories (w. or w/o backorders)
- **multiple products, multiple resources, multiple processes (workforce given)**
  find: all outputs, all inventories, use of processes

Aggregate Planning-Summary

<u>The following scenarios have been discussed:</u>

- **multiple products, multiple workstations (workstation capcities given)**

  find: all sales, all outputs, all inventories (w. or w/o backorders)

- **multiple products, multiple workstations**

  find: all sales, all outputs, all inventories (w. or w/o backorders), OT

- **single product, multiple workstations, one resource**

  find: workforce, all sales, all outputs, all inventories (w. or w/o backorders), OT

Aggregate Planning

- **Work to do:**

- **Examples: 5.7, 5.8abcdef, 5.9abcd, 5.10abcd, 5.16abcd, 5.21, 5.22, 5.29, 5.30**

  Replace capacity columns of table in problem 5.29 with

Month	Machine	Worker
1	1350	19000
2	1270	19000
3	1350	19500

- **Minicase BF SWING II**