On the generalization of constraint programming and boolean satisification solving techniques to schedule a resource-constrained project consisting of multi-mode jobs

Alexander Schnell, Richard F. Hartl

University of Vienna, Department of Business Administration, Chair of Production and Operations Management, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria

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A B S T R A C T
In our paper, we analyze new exact approaches for the multi-mode resource-constrained project scheduling (MRCPSP) problem with the aim of makespan minimization. For the single-mode RCPSP (SRCPSP) recent exact algorithms combine a Branch and Bound algorithm with principles from Constraint Programming (CP) and Boolean Satisfiability Solving (SAT). We extend the above principles for the solution of MRCPSP instances. This generalization is on the one hand achieved on the modeling level. We propose three CP-based formulations of the MRCPSP for the G12 CP platform and the optimization framework SCIP which both provide solution techniques combining CP and SAT principles. For one of the latter we implemented a new global constraint for SCIP, which generalizes the domain propagation and explanation generation principles for renewable resources in the context of multi-mode jobs. Our constraint applies the above principles in a more general way than the existing global constraint in SCIP. We compare our approaches with the state-of-the-art exact algorithm from the literature on MRCPSP instances with 20 and 30 jobs. Our computational experiments show that we can outperform the latter approach on these instances. Furthermore, we are the first to close (find the optimal solution and prove its optimality for) 628 open instances with 50 and 100 jobs from the literature. In addition, we improve the best known lower bound of 2815 instances and the best known upper bound of 151 instances.

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1. Introduction

The multi-mode resource-constrained project scheduling problem (MRCPSP) is a generalization of the single-mode RCPSP (SRCPSP) where an additional mode-assignment step has to be considered. The aim is to find the best mode-assignment for a number of jobs subject to nonrenewable resource constraints such that the optimal schedule for the resulting SRCPSp (if existing) optimizes a specific objective function.

For the SRCPSp recent exact algorithms combine a Branch and Bound (BaB) algorithm with principles from Constraint Programming (CP) and Boolean Satisfiability Solving (SAT) (see [5], [15] and [30]). The idea of the CP-SAT algorithms is to combine the domain propagation processed through global constraints (Apt [3]) with the Conflict Analysis (CA) techniques of a SAT solver (Marques-Silva and Sakallah [19]). Therefore, the different propagators of the global constraints generate explanations, i.e. clauses consisting of Boolean literals, for their domain updates and the detected inconsistencies. The latter explanations are transferred to a SAT solving mechanism. The SAT mechanism constructs a conflict graph based on the explanations of the domain propagators and can possibly deduce nogoods and backjumps via CA.

Roughly speaking, nogoods are valid clauses for a SAT model, like e.g. cutting planes in Mixed-Integer Programming (MIP), which possibly prune branches of the BaB-tree. Backjumps are backtracking moves which lead from the actual node a to a preceding node p whereas d(a) – d(p) > 1 holds for the depth levels d(a) and d(p) in the BaB tree. Moreover, the branching strategy of the underlying BaB-algorithm uses conflict statistics of the literals forming the explanations. In general, the algorithms branch on the variables and values based on the number of conflicts the respective literals were involved in (Moskewicz et al. [21]). For a more detailed introduction to the principles of CP and SAT solving and the possible
The aim of this paper is to provide a generalization of the CP-SAT hybrid for the SRCPSP to the MRCPSP. Exact approaches for the MRCPSP have been summarized and tested by Hartmann and Drexel [12], whereas they conclude that the approach of Sprecher and Drexel [32] is the exact method of choice. The most recent exact algorithm of Zhu et al. [37] outperforms the latter approach. They implemented a Branch-and-Cut procedure with a preprocessing and a heuristic step to generate good upper bounds as an input for their algorithm. A recent survey on heuristic approaches for the MRCPSP and a detailed experimental evaluation is given by Peteghem and Vanhoucke [35]. Their computational experiments show that the scatter search procedure of Peteghem and Vanhoucke [34] produces the best results. In this context, it is also important to mention the approach of Coelho and Vanhoucke [8] as they combine SAT solving techniques with a metaheuristic for the SRCPSP to solve the MRCPSP.

Our contribution is an extension of recent exact approaches combining CP and SAT techniques which are efficient for the SRCPSP to the MRCPSP, more precisely the MRCPSP with standard precedence relations. This extension can be partly achieved on the modeling level. We propose three CP models for the MRCPSP which can be formulated in optimization frameworks that integrate an exact solution approach combining CP, SAT and MIP techniques. Moreover, for one modeling formulation we implemented a new global constraint cumulativemmm specifically tailored to renewable resources in the context of multi-mode jobs. Note that we already successfully generalized and applied recent CP-SAT approaches to the MRCPSP with generalized precedence relations in [26]. The paper at hand can be seen as a predecessor of the latter paper.

In the remainder of the paper, we proceed as follows. In Section 2, we describe the MRCPSP and its computational complexity. Section 3 introduces three problem formulations in optimization frameworks which support the solution by a BaB algorithm integrating CP, SAT and MIP techniques. In Section 4, we describe the principles of our new global constraint cumulativemmm. Section 5 discusses the results of our computational experiments and draws a comparison to the state-of-the-art exact approach of Zhu et al. [37]. The paper ends with a conclusion derived from the obtained results.

2. Problem description and complexity

The MRCPSP is a generalization of the SRCPSP, where every job \( j \in J = \{0, \ldots, n+1\} \) can be processed in different modes \( k \in M_j \subseteq \mathbb{N} \). The jobs 0 and \( n+1 \) are dummy jobs representing the start and the end of the complete project, i.e. in the beginning every job with no predecessor and every job with no successor is connected to the dummy job 0 and \( n+1 \) in the precedence network, respectively. Moreover, the jobs can not be preempted.

Moreover, a set of nonrenewable (resource) demands \( N(R) \subseteq \mathbb{N} \) with a maximal capacity of \( C^r_j, r \in N(C^r_j, r \in R) \) is given. Every job’s integer duration \( d_j \geq 0 \), nonrenewable (renewable) resource consumption \( c^r_j, r \in N(C^r_j, r \in R) \) is dependent on the selected mode \( k \in M_j \).

Nonrenewable resources \( r \in N \) like e.g. a project budget or energy are available for the complete planning horizon. Once job \( j \) is processed in mode \( k \), \( C^r_j - c^r_{jk} \) units of the nonrenewable resource \( r \in N \) are still available for the remaining jobs. Moreover, a constant amount \( C^r_j \) of a renewable resource \( r \in R \) like e.g. a number of machines or workers is available at every point in time.

Furthermore, a job \( j \in J \) cannot end after a job from its successor set \( \Sigma_j \) has started, i.e. in our paper we only consider standard precedence relations. As objective, we consider makespan minimization.

The solution of the MRCPSP can be divided into two steps. The first step consists of finding a feasible mode-assignment w.r.t. the nonrenewable resource capacities. The knapsack problem is polynomially reducible to the latter problem, i.e. already the mode-assignment step is \( \text{NP}-\text{complete} \) for \( |N| \geq 2 \) (Kolisch and Drexel [17]). The second step consists of finding an optimal schedule for a SRCPSP instance, i.e. of finding a schedule which minimizes the makespan and respects the precedence constraints and the renewable resource capacities for a given mode-assignment.\(^1\) Note, that the SRCPSP with the objective of makespan minimization is strongly \( \text{NP}-\text{complete} \) (Blazewicz et al. [6]).

In total, one has to find a feasible mode-assignment at which the minimal makespan of the resulting SRCPSP is not larger than the minimal makespan detected for any other feasible mode-assignment.

As a preprocessing step one can remove redundant nonrenewable resources, inefficient and non-executable modes (see [32] and [111]). Furthermore, lower and upper bounds \( lb(s_j) \) and \( ub(s_j) \) can be deduced for the starting times \( s_j \) by applying forward (backward) recursion [7]. This approach is based on longest path calculations in the precedence network where the arc weights correspond to the minimal mode durations of every job \( j \in J \) w.r.t. the remaining modes. For the evaluation of \( ub(s_j) \), an upper bound \( T \) on the makespan is needed. \( T \) can be given by a problem specific heuristic or \( T_{\text{max}} \) defined in Section 4.

3. CP-models for the MRCPSP

There are two solution frameworks which provide a solution algorithm consisting of a combination of CP, SAT and MIP techniques.

The first is the Constraint Integer Programming framework SCIP, developed by Achterberg [2] and maintained and extended by members of the Zuse Institute in Berlin. SCIP provides a general BaB algorithm for optimization and allows the user to implement plugins, e.g. special branching strategies, primal heuristics and constraint handlers (i.e. global constraints). Moreover, default plugins exist to use SCIP as a stand-alone CP or MIP solver. Furthermore, when the formulated model only consists of default constraint handlers provided by SCIP, the solution algorithm integrates techniques from CP, SAT Solving and MIP.

The second framework is the G12 Constraint Programming Platform [9] provided by the NICTA research team [22]. The user can formulate a problem in the modeling language Zinc [20] and choose between different solution algorithms. Thereby, LCG can also be chosen for the solution of a model. With the G12 Con-

\(^1\) Note that, it can happen that no feasible schedule for the resulting SRCPSP exists, if mode \( m \) has been chosen for job \( j \) and \( c^r_{jm} > C^r_j \).
straint Programming Platform [9] the user is not that flexible as with SCIP but he can solve his model with the state-of-the-art exact algorithm for different variants of the SRCPSP.

In the following, we present three CP-models for the MRCPSP in the above optimization frameworks. Section 3.1 contains one model which can be used within the Zinc-modeling language and Section 3.2 describes two models which can be implemented within SCIP. We also implemented a new constraint handler cumulative for SCIP, which we apply for a model in Section 3.2. The principles of the latter are described in Section 4.

3.1. Formulation for the Zinc-modeling language

For the modeling of the starting time of job i and the mode assignment of job i, we use the integer variables $s_i$ and $x_i$, respectively. With the latter variables and the notation of Section 2, the MRCPSP can be formulated as follows in the CP-modeling language Zinc:

\[
\begin{align*}
\text{min } & s_{n+1} \\
\text{s.t. } & s_i + d_{i,k} \leq s_j, \quad \forall j \in \mathcal{S}_i, \forall i \in J \\
& \sum_{i,j} c_{i,j}^{k,r} \leq C_r, \quad \forall r \in N \\
& \text{cumulative}(s, d, c', C_r), \quad \forall r \in R \\
& s_j \in \{lb(s_j), \ldots, ub(s_j)\}, \quad \forall j \in J \\
& x_j \in M_j, \quad \forall j \in J
\end{align*}
\]

where

- $s = [s_j : j \in J]$
- $d = [d_{i,j} : j \in J]$
- $c' = [c_i^{k,r} : j \in J]$

The dummy job $n + 1$ represents the end of the project. It can only be processed in mode 1 and $d_{n+1,1} = 0$. $c_{i,j}^{k,r}$ and $c_i^{k,r}$ are the durations of job $i$ in mode $k$ and the cumulative duration of job $i$ in mode $k$ respectively.

(2) are multi-mode precedence constraints. With (3) and (4), we assure that the available capacities of the nonrenewable and renewable resources are not exceeded. Hereby, in (4) we use the scheduling specific global constraint cumulative (Baptiste et al. [4]). To apply this constraint for the MRCPSP, we define the variables $s, d$ and $c'$ in (7)-(9).

In the above formulation, variables appear in the indices of parameters, like e.g. in $d_{i,j}$. This modeling technique can only be applied if the respective solver supports the global element-constraint introduced by Hentenryck and Carillon [33].

In general, the element-constraint has the following form:

\[\text{element}(y, x, z)\]

(10) guarantees, that the $y$th element of the variable (or parameter) vector $x$ equals the variable $z$, i.e. $x_y = z$. Clearly, if $x$ has $n$ entries, it must hold that $y \leq n - 1$ if zero is the first index. Propagation algorithms captured by the element-constraint can infer domain updates for the variable $z$ in case of domain updates of $y$ or of the entries $x_i$ of $x$ and vice versa.

In the case of our model, the terms $d_{i,k}$ and $c_i^{k,r}$ are internally transformed to new variables $d'_i$, $c_i^{k,r}$ and $c_i^{k,r}$ by posting the following constraints:

\[\text{element}(x_i, [d_{i,k} : k \in M_i], d'_i), \quad \forall i \in J\]
variables \( s_j \), \( j \neq i \), where the mode-assignment has not been done yet, can be considered for the domain propagation and the inconsistency check of a variable \( s_{i,m} \).

To overcome the disadvantages above, we aimed at implementing a new global constraint \( \text{cumulative}^{\text{mm}} \) for SCIP to be able to apply a more general form of domain propagation and explanation generation for renewable resources in the context of multi-mode jobs where we only have to introduce starting time variables \( s_j \) for every job \( j \in J \). The principles of the \( \text{cumulative}^{\text{mm}} \)-constraint are outlined in Section 4.

With our new constraint and again binary variables \( x_{i,k} \) for the mode-assignment, we formulate the MRCPSP as follows in SCIP:

\[
\min s_{n+1}
\]

\[
s.t. \sum_{k \in K_M} x_{i,k} = 1, \quad \forall i \in J
\]

\[
s_i + \sum_{k \in K_M} d_{i,k} \cdot x_{i,k} \leq s_j, \quad \forall j \in G_i, \forall i \in J
\]

\[
\sum_{i,j,k,r} c_{i,j,k,r}^{\text{p}} \cdot x_{i,k} \leq C^p_r, \quad \forall r \in R
\]

\[
\sum_{j \in J} \sum_{i \in I} \sum_{k \in K_M} \text{cumulative}^{\text{mm}}(s, \bar{x}, \tilde{d}, \tilde{c}, C^p_r), \quad \forall r \in R
\]

\[
s_j \in [\text{lb}(s_j), \ldots, \text{ub}(s_j)], \quad \forall j \in J, \forall k \in M_j
\]

\[
x_{i,k} \in \{0, 1\}, \quad \forall i \in J, \forall k \in M_j
\]

### 4. Principles of the \( \text{cumulative}^{\text{mm}} \)-constraint

With our \( \text{cumulative}^{\text{mm}} \)-constraint, one can model renewable resource constraints for multi-mode jobs. The main ingredients of the \( \text{cumulative}^{\text{mm}} \)-constraint are a feasibility check, constraint propagation and explanation generation (see Sections 4.1 and 4.2) for a certain resource \( r \in R \).

Our constraint enforces feasibility w.r.t. the renewable resource \( r \in R \):

\[
\sum_{j \in J} \sum_{i \in I} x_{i,k,r} \cdot c_{j,m,r} \leq C^p_r, \quad \forall t \in \{1, \ldots, T\}
\]

The other constraints in the SCIP-model of the MRCPSP are modeled by SCIP-intern constraints and can therefore be handled by the SCIP-intern solution principles.

The constraint propagation procedure consists of a redundancy check and a domain reduction step. We firstly check the multi-mode data for redundancy in the current node of the BaB tree. In concrete, if we assume the maximal mode duration, the maximal resource consumption and the maximal processing interval for every job \( j \in J \) in the current node and the underlying schedule is feasible w.r.t. the renewable resource \( r \in R \), we can locally remove our constraint from the solution procedure. This is due to the fact, that in the above case, it cannot be violated anymore in the succeeding branches of the BaB tree.

The domain reduction step is mainly based on the calculation of a minimal problem instance (MPI) [13] in every processed node of the BaB tree, i.e. the transformation of the multi-mode data to a single-mode representative. Therefore, we calculate a minimal processing version \( \text{MPV}_{s_j} = \text{domain}(s_j); d_{\text{min}}, c_{\text{min}}^{\text{mm}} \) for every job \( j \in J \) and renewable resource \( r \in R \) as follows:

\[
d_{\text{min}}^{\text{mm}} = \min_{k \in K_M} \{ d_{i,k} : \text{ub}(x_{i,k}) > 0 \}
\]

\[
c_{\text{min}}^{\text{mm}} = \min_{k \in K_M} \{ c_{i,j,k,r}^{\text{p}} : \text{ub}(x_{i,k}) > 0 \}
\]

In (32) and (33), we calculate the minimal duration and resource consumption of resource \( r \in R \) w.r.t. the modes which have not been excluded \((\text{ub}(x_{i,k}) > 0)\) in the recent node of the BaB tree. With the MPI at hand, we can apply standard constraint propagation algorithms for renewable resources like e.g. timetable propagation (TP) and edge finding [4]. Our current implementation of the \( \text{cumulative}^{\text{mm}} \)-constraint only integrates TP. TP is based on the evaluation and reasoning on the so-called compulsory parts \( cp_j \) of the jobs \( j \in J \) (Schutt et al. [28]):

\[
\text{If} \quad \text{lb}(s_j) + d_{\text{min}}^{\text{mm}} > \text{ub}(s_j) : \quad cp_j = \{ \text{ub}(s_j), \ldots, \text{lb}(s_j) + d_{\text{min}}^{\text{mm}} - 1 \}
\]

\[
\text{Else} \quad cp_j = \emptyset
\]

If \( cp_j \neq \emptyset \), job \( j \) is surely processed at all time points \( t \in cp_j \).

#### Example 4.1 TP for multi-mode jobs

Assume that in the course of the BaB-algorithm of SCIP and after the redundancy check, our constraint propagation procedure has the following input:

\( \text{domain}(s_1), \text{domain}(x_{1,1}), d_{1,1}^{\text{p}}, c_{1,1}^{\text{p}} \) = \((3, 4, 5), \{0, 1\}, 2, 1 \)

\( \text{domain}(s_1), \text{domain}(x_{2,2}), d_{2,1}^{\text{p}}, c_{2,1}^{\text{p}} \) = \((3, 4, 5), \{0, 1\}, 3, 2 \)

\( \text{domain}(s_2), \text{domain}(x_{2,1}), d_{2,1}^{\text{p}}, c_{2,1}^{\text{p}} \) = \((2, 3, 4), \{1\}, 3, 2 \)

The maximal capacity of the renewable resource \( 1, C^p_1 = 2 \). We can see that job 2 is processed in mode 1, as \( x_{2,1} = 1 \). Thus, \( d_{\text{min}}^{\text{mm}} = 3 \) and \( c_{\text{min}}^{\text{mm}} = 2 \). Moreover, job 2 is surely processed at the time point 4, as its compulsory part \( cp_2 = \{4\} \). Next, we consider job 1 with \( d_{\text{min}}^{\text{mm}} = 2 \) and \( c_{\text{min}}^{\text{mm}} = 1 \). \( cp_1 = \emptyset \) but we can deduct a domain update. As \( \text{lb}(s_1) + d_{\text{min}}^{\text{mm}} = 3 + 2 = 5 \), starting job 1 at its lower bound would lead to a resource conflict at the time point 4. The TP algorithm will find the largest time point \( t_1 = 4 = 4 \) such that the capacity is violated (2 + 1 = 2). After that \( \text{lb}(s_1) \) would be updated to \( t_1 = 5 \) which equals \( \text{ub}(s_1) \) and a new compulsory part of job 1 \( cp_1 = \{5, 6\} \) is evaluated.

Note, that the principles of our constraint propagation procedure are standard techniques. These are applied in a similar way in CP solvers like e.g. JaCoP [18] which provide the cumulative-constraint supporting variable durations and resource consumptions for every job.

The idea of integrating explanation generation, i.e. of processing reasons for the deduced domain reductions or inconsistencies to a SAT solving mechanism is rather new. To our knowledge, there are only two optimization frameworks integrating this feature, i.e. SCIP and the G12 Constraint Programming Platform [9]. Schutt [27] describes principles for explanation generation in the context of jobs having variable durations and resource consumptions. These explanation generation techniques are integrated in the cumulative-constraint provided by the G12 Constraint Programming Platform [9]. In our \( \text{cumulative}^{\text{mm}} \)-constraint, we explain the reasons for the domain reductions or inconsistencies in a different way. In the next two sections we introduce our strategy for explanation generation and compare it to the strategy of Schutt [27].

#### 4.1. Explanations for timetable propagation with multi-mode jobs

In order to integrate our constraint into the SCIP-intern CA mechanism, we have to provide functions for the \( \text{cumulative}^{\text{mm}} \)-constraint which derive explanations for the inconsistencies or domain updates detected by the TP algorithm. These explanations can be seen as clauses consisting of boolean literals of the form:

\( \{ \| s_j \leq t \|, \| s_j \geq t \| : t = 0, \ldots, T - 1 \} \)

and

\( \{ \| x_{i,k} = 0 \|, \| x_{i,k} = 1 \| \} \)
Assume now, the TP algorithm found an inconsistency because of the jobs having their compulsory parts \( c_p_j \) (see (34)) at time \( t_e \) and cause a resource violation:

\[
\sum_{j \in c_p_j} d_{j,r}^{\min} > c^{p}_r
\]

Then, our constraint handler derives the following explanation:

\[
\bigwedge_{j \in c_p_j} EXP_j \implies \text{false} \tag{35}
\]

(35) can be divided into the subexplanations \( EXP_j \) for every job participating in the conflict:

\[
EXP_j = \left( [t_e - d_{j,r}^{\min} + 1 \leq s_j] \land [s_j \leq t_e] \right) \bigwedge_{m:\text{mode} \in M, \text{and } \text{ub}(x_{j,m})=0} \left[ [x_{j,m} == 0] \right] \tag{36}
\]

\( EXP_j \) is correct as in a situation, where the variables \( s_j \) and \( x_{j,m} \) have bounds as in (36), the compulsory parts \( c_p_j \) of the involved jobs would again include the timepoint \( t_e \) and this would again lead to a resource violation. Note, that \( d_{j,r}^{\min} \) and \( c^{p}_j \) are calculated by (32) and (33), respectively.

In addition to explanations for inconsistencies, our constraint handler also processes explanations for domain updates deduced by our TP algorithm to the SCIP-intern CA mechanism. SCIP stores information about the time the bound changes took place through a so-called bound change index (BCI) and about the constraints which processed the domain updates. In the course of the BCI, the SCIP-intern CA can ask our constraint handler for an explanation of the new lower bound \( lb(s_j) \) of job \( j \) at the BCI \( b \), if it was deduced by the cumulative-mmm-constraint.

Therefore, we introduce the timepoint \( t = lb(s_j) - 1 \). Jobs \( j \in f(i) \) with compulsory parts \( c^{p}_j \) where \( t \in c^{p}_j \) such that:

\[
c^{p}_j + \sum_{j \in c_p_j} d_{j,r}^{\min} > c^{p}_j
\]

were responsible for the bound change at the BCI \( b \). Additionally the lower bound \( lb(s_j) \) of job \( i \) has to exceed a certain value for the domain update. The complete explanation consists of two clauses \( e_i \) and \( f \), where \( e_i \) contains the minimal lower bound of \( s_j \) and \( f \) the compulsory parts \( c^{p}_j \) of jobs \( j \neq i \) with \( t \in c^{p}_j \). \( e_i \) is given as follows:

\[
e_i = \left[ [t - d_{j,r}^{\min} + 1 \leq s_j] \right] \bigwedge_{m:\text{mode} \in M, \text{and } \text{ub}(x_{j,m})=0} \left[ [x_{j,m} == 0] \right]
\]

The clause \( f \) is as follows:

\[
f = \bigwedge_{j \neq i \text{ and } t \notin c^{p}_j} \bigwedge_{j \in c_p_j} \left( EXP_j \right)
\]

where \( EXP_j \) is given through a small variation of (36). We replace \( t_e \) by \( t \), \( d_{j,r}^{\min} \) by \( d_{j,r}^{\min} \) and \( \text{ub}(x_{j,m}) \) by \( \text{ub}(x_{j,m}) \). Thus, we evaluate the latter values for the given BCI \( b \). The complete explanation for the bound change \( lb(s_j) \rightarrow lb(s_j) \) is given through:

\[
e_i \land f \implies lb(s_j) \leq s_j \tag{37}
\]

The argument for the correctness of the above explanation is the same as in the inconsistency case. Note, that the explanation (37) depends on the BCI \( b \).

In our TP algorithm we process the bound changes in a pointwise manner, i.e. we guarantee that \( lb(s_j) \rightarrow lb(s_j) \leq d_{j}^{\min} \). With this, we want to imitate the pointwise explanations proposed by Schutt et al. [28]. The explanations for the upper bound changes are processed in a symmetric way.

SCIP generates a conflict graph based on the explanations of our constraint handler and the explanation generators of the other constraints and can possibly deduce nogoods and backjumps for the following branches in the BaB algorithm. If a constraint propagation algorithm leads to many domain reductions, CA can be very efficient for the complete solution procedure. An example for the CA process, i.e. for nogood generation on a conflict graph can be found in Schutt et al. [31].

The following example illustrates a possible outcome of our explanation generation procedure.

**Example 4.2.** Firstly, we extend Example 4.1 by another job with two modes and the following input:

\[
\begin{align*}
\text{domain}(s_1), \text{domain}(x_{3,1}), d_{3,1}, c^{s}_{3,1,1} = & (15), (1), 2, 2) \\
\text{Note, that there is a resource conflict at time point } t_e = 6, \text{as } 6 \in c_{p_1} \land c_{p_2} \text{ and } c^{s}_{3,1,1}(= 2) + c^{s}_{3,1,1}(= 1) > 2. \text{ The explanation for this inconsistency is as follows:} \\
\langle 5 \leq s_1 \rangle \land \langle 5 \leq s_3 \rangle \land \langle 5 \leq s_6 \rangle \land \langle 5 \leq x_{3,2} \rangle \rightarrow \text{false} \tag{38}
\end{align*}
\]

Note, that job 3 is processed in mode 1 and for job 1 it holds, that \( \text{ub}(x_{3,1}) > 0, \forall k \in M \). After the initialization of the SCIP-intern CA, SCIP asks our constraint handler for the reason of the lower bound change of \( s_1 \), from 3 to 5 from Example 4.1, i.e. an explanation for the literal \( [5 \leq s_1] \).

Our constraint handler gives the following explanation:

\[
\langle 3 \leq s_1 \rangle \land \langle 3 \leq s_3 \rangle \land \langle 3 \leq s_6 \rangle \land \langle 3 \leq x_{3,2} \rangle \rightarrow \langle 5 \leq s_1 \rangle \tag{39}
\]

Every boolean literal from (38) and (39) is added as a new node to the SCIP-intern conflict graph. Moreover, an arc is constructed from every boolean literal of the left-hand side of the explanation to the boolean literal on the right-hand side.

4.2. Comparison to other explanation generation techniques and possible improvements

Schutt [27, p.169] also introduces explanations for the cumulative-constraint where the durations and the resource consumptions of the jobs can be variables. In our G12-model of Section 3, we use this constraint with the duration vector \( d \) and the resource consumption vector \( c_r \) to model the resource constraint for the renewable resource \( r \in R \). After the transformation given by (11) and (12), the G12 solution approach will only use the variable vectors \( d \) and \( c_r \) in the cumulative-constraint. These are connected to the original durations and resource consumptions by the element-constraint (see [11] and [12]). With our notation, the preliminary version of the explanations for a lower bound update of \( s_j \) to \( lb(s_j) \) applied in the cumulative-constraint (see [27, p.169]) are as follows (at the BCI \( b \)):

\[
\begin{align*}
\left[ lb^{r}(s_j) - lb^{r}(d_i') \leq s_j \land \|lb^{r}(d_i')\| \leq c_{r}^{j} \right] & \implies \left[ lb^{r}(s_j) - lb^{r}(d_i') \leq s_j \land \|lb^{r}(d_i')\| \leq c_{r}^{j} \right] \land \\
\left[ lb^{r}(s_j) - lb^{r}(d_i') \leq s_j \land \|lb^{r}(d_i')\| \leq c_{r}^{j} \right] & \implies \left[ lb^{r}(s_j) - lb^{r}(d_i') \leq s_j \land \|lb^{r}(d_i')\| \leq c_{r}^{j} \right]
\end{align*}
\]

(40)

Schutt [27] notes, that these explanations can be strengthened by choosing different values \( q_i \) and \( l_i \) instead of \( lb^{r}(d_i') \) and \( lb^{r}(c_{r}^{j}) \), respectively. For example consider the case where the domain of \( d_i' \) is internally encoded as a range of consecutive values but the set \( D_i = \{d_{i,m}, m \in M\} \) consists of nonconsecutive values and it holds that \( lb^{r}(d_i') \notin D_i \). Then, by using \( q = \min \{d_{i,m} : d_{i,m} > lb^{r}(d_i')\} \), the explanation (40) can be strengthened. Schutt [27] specifies the values \( q_i \) and \( l_i \) which lead to the strongest explanations.
In our explanation for a domain update (see (37)), we omit the part where resource consumptions of the involved jobs are explained as in (40). This is due to the fact, that as soon as certain modes are excluded, we can reason about the minimal duration \(d_j^{\text{min}}\) and the minimal resource consumption \(c_j^{\text{min}}\). Thus we do not have to introduce explanations for both durations and resource consumptions. This can be an advantage compared to the explanations of Schutt [27].

Assume therefore, that we are in a situation in a node \(c\) of the BaB tree where the input is the same for both algorithms, and both algorithms already generated the explanations (37) and (40), respectively at node \(b\) with the same set of jobs \(J_{\text{exp}}\) involved in both explanations. Additionally,
\[
d_j^{\text{min,b}} = lb_b(d_j^i), \quad \forall j \in J_{\text{exp}}
\]
\[
c_j^{\text{min,b}} = lb_b(c_j^i), \quad \forall j \in J_{\text{exp}}
\]

Moreover, the left hand side of (37) is true at node \(c\) and
\[
\exists k \in J_{\text{exp}} : \quad t_b \in c p_k \land lb_b(c_j^i) > lb_b(c_j^i_k)
\]
whereat the first two lines of (40) are true. As (37) is true, our algorithm will immediately deduce \(lb_b(s_j) \leq s_j\).

Because of (43), the G12-algorithm will not immediately deduce the latter lower bound update. As the first two lines of (40) are true and (41) and (42) hold, we can conclude that
\[
x_j \in M_j^b = \{ m : m \in M_j \land ub(x_{j,m}) > 0 \}, \quad \forall j \in J_{\text{exp}}
\]
and thus \(lb_b(c_j^i) \leq c_j^i\). i.e. \(lb_b(c_j^i)\) can be updated to \(lb_b(c_j^i)\). Now, the complete left hand side of (40) is true and the G12-algorithm will also deduce \(lb_b(s_j) \leq s_j\).

The update of \(lb_b(x_j)\) outlined in (44) and the update of \(lb_b(c_j^i)\) have to be processed by the element-constraints (11) and (12) in the G12-algorithm before the explanation (40) leads to the update of \(lb_b(s_j)\). As this update happens immediately with our explanation (37), there are cases where our explanation generation strategy can lead to time savings.

Our explanations can also be strengthened.

**Example 4.3.** Consider a job 1 with the following input at the current node:

\[
\begin{align*}
\text{(domain}(x_{1,1}), d_{1,1}, c_{1,1}^{d_{1,1}}) &= (\{0\}, 2, 2) \\
\text{(domain}(x_{1,2}), d_{1,2}, c_{1,2}^{d_{1,2}}) &= (\{0\}, 3, 3) \\
\text{(domain}(x_{1,3}), d_{1,3}, c_{1,3}^{d_{1,3}}) &= (\{1\}, 4, 3)
\end{align*}
\]

Assume that our TP algorithm would detect an inconsistency at the time point 4 and job 1 is involved in the latter inconsistency, i.e. 4 \(\in c p_1\). As \(d_j^{\text{min}} = 4\), the part of the explanation containing job 1 is as follows:

\[
\llbrace 1 \leq s_1 \rrbrace \land \llbrace s_1 \leq 4 \rrbrace \land \llbrace x_{1,1} == 0 \rrbrace \land \llbrace x_{1,2} == 0 \rrbrace
\]

If the global domain of \(s_1\) equals \([2, \ldots, 6]\), we can strengthen the explanation as

\[
\llbrace 1 \leq s_1 \rrbrace \text{ is globally true,}
\]
\[
\llbrace x_{1,1} == 0 \rrbrace \land \llbrace x_{1,2} == 0 \rrbrace \implies \llbrace x_{1,1} == 0 \rrbrace,
\]
\[
\llbrace x_{1,1} == 0 \rrbrace \land \llbrace s_1 \leq 4 \rrbrace \implies 4 \in c p_1,
\]
\[
c_{1,2,1}^d = c_{1,3,1}^d.
\]

Thus, we can use the following stronger explanation for the compulsory part of job 1:

\[
\llbrace s_1 \leq 4 \rrbrace \land \llbrace x_{1,1} == 0 \rrbrace
\]

Motivated by the above example, assume that job \(i\) is part of the job set \(J_{\text{exp}}\) involved in the explanation (37) w.r.t. time point \(t\). Moreover, let \(lb_b(s_j)\) be its global lower bound. We can possibly strengthen the explanation (37) by strengthening the part of the explanation integrating job \(i \in J_{\text{exp}}\).

This can done in two steps:

Firstly, if \(t - d_j^{\text{min}} + 1 < lb_b(s_i)\), omit \(\llbrace t - d_j^{\text{min}} + 1 \leq s_i \rrbrace\).

Set \(d_j^{\text{min}} := \min\{d_j^{\text{min}}, t - lb_b(s_i) + 1\}\).

Secondly, we determine the set \(B_i^j\) consisting of the modes \(m \in M_i\) which fulfill:

\[
d_{i,m} \geq d_j^{\text{min}}
\]

\[
c_{i,m}^d + \sum_{j \in J_{\text{exp}} \setminus \{i\}} c_{j,m}^{\text{min}} > c_j^d
\]

Now, we can substitute

\[
\llbrace m : m \in M_i \land ub(x_{i,m}) = 0 \rrbrace
\]

by

\[
\llbrace m : m \in M_i \setminus B_i^j \rrbrace
\]

Because of the evaluation of \(d_j^{\text{min}}\) and \(c_j^{\text{min}}\) in (32) and (33) and because of (45) and (46) it holds that

\[
M_i \setminus B_i^j \subseteq \{ m : m \in M_i \land ub(x_{i,m}) = 0 \}
\]

Thus, the part of the explanation (37) integrating job \(i\) is possibly stronger. Finally, we update \(c_{j,m}^d := \min\{c_{j,m}^d, m \in B_i^j\}\). After the latter update, we continue with the next non-processed job.

### 5. Computational experiments

The three CP models from Section 3 were solved on the Vienna Scientific Cluster (VSC). Thereby, the cluster nodes integrate a X86-64 architecture running under Red Hat/Linux with two six-core Intel Westmere X5650 processors of 2.66GHz and with 24GB RAM. For the solution of the models from Section 3.1, we used the G12 Constraint Programming Platform [9] 2.0.0 provided by the NICTA research team [22]. Thereby, we formulated the models in Zinc and solved them by the LCG-plugin g12_fdx. For the implementation of the constraint handler cumulative of the formulation and solution of the SCIP-models of Section 3.2, we used SCIP 3.1.0 in combination with the programming languages C/C++. We set the parameters in SCIP such that feasibility is detected fast (with SCIP_PARAMEMPHASIS_FEASIBILITY). Furthermore, we impose a memory limit of 2GB RAM for instances with less than 100 jobs and of 3GB for the 100-job instances.

The three CP-models are denoted by the following abbreviations:

**G12** The model from Section 3.1 formulated in Zinc.

**SCIPopt** The SCIP-model of Section 3.2 integrating the existing **optcumulative-constraint**.

**SCIP** The SCIP-model of Section 3.2 integrating our **cumulative-constraint**.

Moreover, for every model we distinguish two solution approaches which differ in the generation of the initial domains:

**Max** The initial domains of the starting time variables are evaluated by forward (backward) recursion based on the trivial upper bound \(T_{\text{max}} = \sum_{j \in J} d_j^{\text{max}}\), where \(d_j^{\text{max}}\) is the maximal mode duration of job \(j\).

**Best** The initial domains are generated based on twelve different upper bounds \(T_l, \ldots, T_{12}\) where \(T_l\) equals the best known upper bound from the literature and \(T_l = T_{l-1} + 4, \forall l = 2, \ldots, 12\). A model is run on the processor \(l = 1, \ldots, 12\) with initial domains based on \(T_l\). Hence, in this
case we apply a parallel approach which applies twelve processors. In the end, we take the best results w.r.t. all twelve processors.

In total, we compare six different solution approaches to the state-of-the-art exact approach (Branch-and-Cut) for the MRCPSP (MMBAC) of Zhu et al. [37]. In addition, the average deviation from the critical path lower bound is compared to the one obtained by the best metaheuristic for this problem (VANP14) [35].

The test instances are from the PSPLIB (Kolisch and Sprecher [18]) and from the instance sets MMLIB and MMLIB+ generated by Peteghem and Vanhoucke [35] (see also [36]). In total, we consider instances with 20, 30, 50 and 100 jobs, whereas we abut the solution process after a time limit of 180 s, 360 s, 5400 s and 7200 s, respectively. To assure a fair comparison to the approach of Zhu et al. [37], we used time limits which are approx. 10% of their time limits for the 20- and 30-job instances. The factor 10 is based on the values of our processor and their processor (Intel Westmere X5650 with 2.66GHz and Xeon with 1.80 Ghz from 2004, respectively) in the Passmark CPU benchmark [25]. Moreover, we also use the comparison factor of approx. 0.68 based on the clock rates of both CPUs for further experiments. This leads to time limits of 1200 s and 2400 s.

We compare the different approaches based on the following measurements:

<table>
<thead>
<tr>
<th>#feas</th>
<th>#opt</th>
<th>#best</th>
<th>t_avg</th>
<th>Gopt(%)</th>
<th>Gopt(%)</th>
<th>I_Lb</th>
<th>I_ub</th>
</tr>
</thead>
<tbody>
<tr>
<td>554</td>
<td>554</td>
<td>554</td>
<td>11.73</td>
<td>0.12</td>
<td>0.06</td>
<td>30</td>
<td>0.32</td>
</tr>
<tr>
<td>554</td>
<td>554</td>
<td>554</td>
<td>5.2</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>554</td>
<td>554</td>
<td>554</td>
<td>12.33</td>
<td>1.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>554</td>
<td>554</td>
<td>554</td>
<td>9.06</td>
<td>0.87</td>
<td></td>
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<td>554</td>
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<td>554</td>
<td>554</td>
<td>22.29</td>
<td>4.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>554</td>
<td>554</td>
<td>554</td>
<td>20.58</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>554</td>
<td>554</td>
<td>554</td>
<td>10.39</td>
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<td></td>
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<td>554</td>
<td>554</td>
<td>554</td>
<td>33.96</td>
<td>0.32</td>
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<td>554</td>
<td>554</td>
<td>554</td>
<td>130.37</td>
<td>3.16</td>
<td></td>
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</tr>
<tr>
<td>554</td>
<td>554</td>
<td>554</td>
<td>113.66</td>
<td>2.85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>554</td>
<td>554</td>
<td>32.06</td>
<td>0</td>
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<td></td>
<td></td>
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<td>554</td>
<td>554</td>
<td>554</td>
<td>0</td>
<td>0.32</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that optimality gaps for the G12 approaches are calculated w.r.t. the lower bounds computed by the best SCIP approach. This is due to the fact that we did not find a way to receive these values from the G12 framework. Therefore, we also omit the results concerning the lower bound improvements for the G12 approaches.

Table 1 shows the results for our models and the state-of-the-art exact approach of Zhu et al. [37] (MMBAC) on the 554 feasible 20-job instances from the PSPLIB. For these instances MMBAC outperforms all of our approaches for the small time limits as they can solve all feasible instances to optimality. The same holds for the single core approaches (G12Max, SCIPMax and SCIPoptMax) when applying the larger time limit. Nevertheless, G12Max is highly competitive to the approach of Zhu et al. [37] in this scenario. The parallel approach G12Best outperforms the state-of-the-art exact approach from the literature for the larger time limits. We can also solve all feasible instances to optimality, but MMBAC is two times slower, when taking into account the clock rates of both processors.

Furthermore, the SCIP-approaches using our cumulative constraint significantly outperform the SCIP-approaches integrating the existing optcumulative constraint. With SCIPBest we can solve 40 more instances to optimality in an approximately five times lower average solution time compared to SCIPoptBest for the larger time limits. Moreover, SCIPBest is competitive to MMBAC and G12Best in this scenario. Note that within a time limit of approximately 3806 s the approach SCIPBest can solve all 554 instances to optimality. The average solution time is 28.47 s in the latter case.

Furthermore, for the large time limit, the G12 approaches and the approach SCIPBest produce a better optimality gap than the best metaheuristic presented by Peteghem and Vanhoucke [35].

Table 2 shows the results for the 552 feasible 30-job instances from the PSPLIB. Again, considering the small time limits MMBAC outperforms all of our approaches. However, we can already solve 9 more instances to optimality than MMBAC with the single-core approach G12Max for the larger time limits. Moreover, in this situation G12Max is approx. 1.23 times faster than MMBAC, again taking into account the clock rates of both processors. Furthermore, the parallel approach G12Best significantly outperforms MMBAC both regarding average solution times and solution quality in this scenario. Again, SCIPBest (SCIPMax) is considerably better than SCIPoptBest (SCIPoptMax). We can solve 36 (44) more in-

---

2 Our evaluations concerning the best known upper bounds are based on the state of this website on the 28.09.2016.

3 Note that in this case, we can feasibly make this comparison as Peteghem and Vanhoucke [35] use the gap of their best makespan w.r.t. the known optimal solution of an instance for these instances.
Table 2
Results on the 30-job instances.

<table>
<thead>
<tr>
<th></th>
<th>#feas</th>
<th>#opt</th>
<th>#best</th>
<th>t_{tot} (s)</th>
<th>C_{opt}(%)</th>
<th>G_{gap}(%)</th>
<th>I_{ub}</th>
<th>I_{lb}</th>
</tr>
</thead>
<tbody>
<tr>
<td>G12Max</td>
<td>552</td>
<td>466</td>
<td>480</td>
<td>91.2</td>
<td>14.3</td>
<td>14.3</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>G12Best</td>
<td>552</td>
<td>495</td>
<td>514</td>
<td>58.86</td>
<td>3.01</td>
<td>12.96</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SCIPMax</td>
<td>552</td>
<td>486</td>
<td>492</td>
<td>53.06</td>
<td>4.72</td>
<td>14.11</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SCIPBest</td>
<td>552</td>
<td>494</td>
<td>507</td>
<td>46.0</td>
<td>3.87</td>
<td>13.39</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SCIPoptMax</td>
<td>552</td>
<td>440</td>
<td>451</td>
<td>84.64</td>
<td>10.82</td>
<td>15.60</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>SCIPoptBest</td>
<td>552</td>
<td>454</td>
<td>473</td>
<td>72.69</td>
<td>8.86</td>
<td>14.10</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3
Results on the 50-job instances from MMLIB.

<table>
<thead>
<tr>
<th></th>
<th>#feas</th>
<th>#opt</th>
<th>#best</th>
<th>t_{tot} (s)</th>
<th>C_{opt}(%)</th>
<th>G_{gap}(%)</th>
<th>I_{ub}</th>
<th>I_{lb}</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCIPMax</td>
<td>540</td>
<td>405</td>
<td>415</td>
<td>1409.69</td>
<td>17.73</td>
<td>17.86</td>
<td>276</td>
<td>-</td>
</tr>
<tr>
<td>SCIPBest</td>
<td>540</td>
<td>420</td>
<td>440</td>
<td>1252.25</td>
<td>9.86</td>
<td>9.88</td>
<td>280</td>
<td>-</td>
</tr>
<tr>
<td>G12Max</td>
<td>532</td>
<td>363</td>
<td>377</td>
<td>1952.36</td>
<td>11.21</td>
<td>27.61</td>
<td>19</td>
<td>-</td>
</tr>
<tr>
<td>G12Best</td>
<td>539</td>
<td>367</td>
<td>413</td>
<td>1861.9</td>
<td>8.87</td>
<td>25.34</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>VAN14</td>
<td>540</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>23.79</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4
Results on the 100-job instances from MMLIB.

<table>
<thead>
<tr>
<th></th>
<th>#feas</th>
<th>#opt</th>
<th>#best</th>
<th>t_{tot} (s)</th>
<th>C_{opt}(%)</th>
<th>G_{gap}(%)</th>
<th>I_{ub}</th>
<th>I_{lb}</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCIPMax</td>
<td>518</td>
<td>312</td>
<td>322</td>
<td>3127.93</td>
<td>341.33</td>
<td>164.09</td>
<td>18</td>
<td>213</td>
</tr>
<tr>
<td>SCIPBest</td>
<td>535</td>
<td>338</td>
<td>348</td>
<td>2740.31</td>
<td>33.71</td>
<td>27.13</td>
<td>23</td>
<td>217</td>
</tr>
<tr>
<td>G12Max</td>
<td>219</td>
<td>150</td>
<td>154</td>
<td>5963.62</td>
<td>21.35</td>
<td>25.71</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>G12Best</td>
<td>404</td>
<td>245</td>
<td>250</td>
<td>5255.9</td>
<td>16.91</td>
<td>21.29</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>VAN14</td>
<td>540</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>24.02</td>
<td>21.51</td>
</tr>
</tbody>
</table>

As the Branch-and-Cut approach of Zhu et al. [37] is based on a MIP formulation of the MRCPSP, it is highly dependent of a starting solution with a small makespan to reduce the number of binary variables. They use starting solutions computed by a problem specific heuristic whose makespan on average only deviates by 2.18% from the best known makespans of the PSPLIB. An advantage of our SCIP- and G12 approaches is that they still produce competitive results with the relatively high upper bound \( t_{\text{max}} \) as input.

Moreover, the input upper bounds leading to the best results in the parallel approach G12Best deviate on average by 19.69% and 22.14% from the best known upper bounds from the literature for the 20- and 30-job instances, respectively.

Furthermore, for this instance set, all of our solution approaches except SCIPopt exhibit a better gap to the critical path lower bound than the best metaheuristic presented by Peteghem and Vanhoucke [35] for the larger time limits.4

Now, we present our results for new instances with 50 and 100 jobs generated by Peteghem and Vanhoucke [35]. To our knowledge, these have not been solved exactly before.

Tables 3 and 4 contain the results for the runs with the 540 feasible 50-job and 540 feasible 100-job instances from the MMLIB, respectively. For these instance sets, the approaches integrating the best SCIP model significantly outperform the approaches integrating the G12-model.

The G12 approaches only produce better average optimality gaps and gaps w.r.t. the critical path lower bound, also if the average value is computed w.r.t. to the instances where both the SCIP and the respective G12 approaches, can compute a feasible solution. This can be observed when regarding the values in brackets near the gap measurements in the SCIP approaches.5

The performance difference of the G12 approaches compared to our best SCIP approaches for the 20- and 30-job instances and the 50- and 100-job instances can be explained in the following way. In the G12 formulation, we use variables as indices of parameter vectors as a concept to model the mode assignment. The G12 solver internally transforms these constructs to a number of element-constraints (see Section 3.1). In this context, \(|J| \cdot (1 + |R| + |N|)\) new variables \(d'_j\), \(c'^c_i\) and \(c'^d_i\) and constraints (see (12), (11) and (13)) have to be introduced. In general, the BaB algorithm has to consider all of these additional variables and constraints in the course of its internal branching, constraint propagation and CA mechanisms. One can see that this solver-internal transformation still works in an efficient way for the 20- and 30-job instances. However, for the 50- and 100-job instances the additional introduction of new variables and constraints leads to a great loss of efficiency of the overall procedure. In the SCIP formulation, we do not need a transformation of the original model. More precisely,

4 However, one should always keep in mind that this metaheuristic is significantly faster in practice than our exact approach.

5 Note that the value in brackets for VAN14 corresponds to the gap w.r.t. the instances where the approach SCIPBest can find a feasible solution.
we tackle the mode assignment by binary variables and the internal handling of cumulative. Obviously, this leads to a significantly more efficient behavior of the overall solution procedure for the 50- and 100-job instances.

A hint to the observation that our best SCIP algorithms (SCIPMax and SCIPBest) should work in a more effective way than the respective G12 algorithms was already given in Section 4.2. A reason why we cannot observe the latter for the 20- and 30-job instances can also be the additional overhead of the SCIP implementation. The SCIP internal algorithm captures a wide variety of additional techniques, e.g. specialized MIP concepts, which only have a small contribution to the overall solution procedure for our special problem. This overhead does not occur in the G12 algorithm. Hence, this additional computational effort can lead to the weaker performance of our best SCIP algorithms compared to the G12 algorithms for the 20- and 30-job instances. For the 50- and 100-job instances the impact of this overhead is of lesser extent and the above assumption about the more efficient behavior of our best SCIP approaches is computationally supported.

In total, we can solve approx. 78% of the 50-job instances and 63% of the 100-job instances to optimality within the given time limits. Moreover, we improved the best known makespans of 18 instances with 50 jobs and 23 instances with 100 jobs. Furthermore, we can improve the lower bounds for approx. 52% of the 50-job instances from MMLIB on average by approx. 28%. For 40% of the 100-job instances from MMLIB tighter lower bounds could be computed. These are on average approx. 23% better than the best known lower bounds. Furthermore, the input upper bounds leading to the best results in the parallel approach SCIPBest are on average 37.41% and 131.20% higher than best known makespans for the 50- and 100-job instances, respectively. This is again an indication that our approaches can also produce good results with relatively high upper bounds as input.

Note that in case of the 50-job instances, we found out that at least 65% of the heuristic solutions given by Peteghem and Vanhoucke [35] correspond to an optimal solution. For the 100-job instances, this holds for at least 47%. Furthermore, the gap to the critical path lower bound computed by the approach SCIPBest is competitive compared to the metaheuristic VANP14 for the 50-job instances. However, for the 100-job instances our SCIP approaches reach their limits w.r.t. producing high quality upper bounds in total for all instances.

To test the influence of the five different project parameters based on which the new data sets of MMLIB were generated (see [35] and [36]), i.e. the order strength (OS), and the renewable and nonrenewable resource strength and resource factor (RS\(p^{(iv)}\) and \(RP^{(v)}\)), on the performance of our best solution approaches SCIPBest (measured by the optimality gap), we conducted a multiple linear regression analysis similar to Peteghem and Vanhoucke [35] for the 50- and 100-job instances of MMLIB. For the 50-job instances, the parameters –0.188, –0.561 and 0.379 are significant coefficients (confidence level of 1%) for OS, \(RS^p\) and \(RP^p\) in the multiple linear regression model, respectively. Moreover, the constant 0.168 is also significant. The coefficient of determination \(R^2\) is approx. 0.66. Thus, for these instance, we can observe that with an increasing number of precedence relations, the performance of the approach SCIPBest increases. This can be explained by the fact that the size of solution space tendentially decreases with an increasing number of precedence relations. Furthermore, the less scarce the renewable resources become, the better is the performance of SCIPBest. Finally, one can observe, that the performance of SCIPBest decreases when the average resource consumption of the multi-mode jobs is increased. For the 100-job instances only the coefficients –17.61 and 11.60 are significant for \(RS^p\) and \(RP^p\), respectively. Tables 5 and 6 show the results for the different parameter groups which correspond to significant coefficients for the 50- and 100-job instances, respectively.

Finally, Table 7 shows the results of the approaches integrating our best SCIP-model for the renownedly more complex 1620 feasible 50-job and 1620 feasible 100-job instances from the instance set MMLIB+. As instances of MMLIB+ on average integrate more nonrenewable and renewable resources than those of MMLIB, one can assume that the performance of the G12 solution approaches is even worse for these instances. Therefore, we only consider the approaches integrating our best SCIP model for these evaluations. For this instance set 2318 lower bounds are on average improved by approx. 51%. Furthermore, the input upper bounds leading to the best results with SCIPBest deviate on average by 555.21% from the best known makespans for MMLIB+.

Note that the value in brackets near \(C_{opt}\) for VANP14 corresponds to the gap to the critical path lower bound for the instances where SCIPBest can find a feasible solution.

A file with more detailed results for every examined instance set is provided as supplementary data to this paper.

A disadvantage of our approaches is that not for every instance of MMLIB100 and of MMLIB+ a feasible solution can be found. This disadvantage can be overcome by running a heuristic (e.g. the one of [35] with a limit of 5000 schedules) in the beginning to obtain a feasible initial solution for every instance as input for our solution approaches. Furthermore, in this context, the influence of scheduling specific primal heuristics integrated into the SCIP framework on our solution approaches would be an interesting future research topic.

### 6. Conclusion

In our paper, we introduced a generalization of the exact CP-SAT approaches for the SRCSP to the MRCPSP. This generalization can on the one hand be achieved on the modeling level. We introduced formulations of the MRCPSP in optimization frameworks (G12 and SCIP) which integrate a BaB-algorithm in combination with CP and SAT techniques. One formulation is usable via the Zinc modeling language which supports the solution of models by LCG, the state-of-the-art exact approach for variants of the SRCSP. Moreover, we proposed two formulations for the optimization framework SCIP. One of the latter is based on our new constraint handler called cumulative and the other one on the
existing optcumulative-constraint introduced by Heinz et al. [14].

The computational experiments show that for the 20- and 30-
job instances at least one of our proposed approaches can outper-
form the state-of-the-art exact algorithm of Zhu et al. [37] for
the MRCPSp when the approaches are compared based on the clock
rates of the CPUs on which they are run. In this situation, our
parallel G12-algorithm is almost two times faster than their ap-
proach on the 20-job instances. Moreover, on the 30-job instances
already our single-core approach using a trivial upper bound as in-
put for the G12-algorithm can solve 9 more instances to optimal-
ity and is approx. 123 times faster. A clear advantage of our ap-
proaches is that they also produce competitive results when large
upper bounds are used as input. In contrast, the quality of the ap-
proach of Zhu et al. [37] is highly dependent on small UBs as in-
put to reduce the initial number of variables.

Moreover, the SCIP-approaches which apply our cumulativemm-constraint are significantly better than the SCIP-
approaches integrating the existing optcumulative-constraint.
For the 20- and 30-job instances, we could solve 40 and 36 more
problem instances to optimality in approximately five and two
times smaller average solution times, respectively.

Finally, we are the first to exactly solve new MRCPSp instances
with 50 and 100 jobs from the literature [35]. On these instance
sets, our SCIP-approaches with cumulativemm significantly out-
perform the G12-approaches. In total, we close (find the optimal
solution and prove its optimality for) 628 open instances with 50-
and 100-jobs from the literature [35]. Moreover, we improve the
best known makespans reported Online [24] for 151 of these in-
stances. In addition we improve the best known lower bound of
2815 instances on average by approx. 46%.

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Supplementary material

Supplementary material associated with this article can be found,

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