
Homework 2

do not forget plot a rate diagram for every example!!

>> Gas Station

In a gas station there is one gas pump. Cars arrive at the gas station according to a Poisson process. The arrival rate is 20 cars per hour. An arriving car finding n cars at the station immediately leaves with probability $q_n = n/4$, and joins the queue with probability $1 - q_n$, $n = 0,1,2,3,4$. Cars are served in order of arrival. The service time (i.e. the time needed for pumping and paying) is exponential. The mean service time is 3 minutes.

1. Determine the stationary distribution of the number of cars at the gas station.
2. Determine the mean number of cars at the gas station.
3. Determine the mean sojourn time (waiting time plus service time) of cars deciding to take gas at the station.

>> TV repair

A repair man fixes broken televisions. The repair time is exponentially distributed with a mean of 30 minutes. Broken televisions arrive at his repair shop according to a Poisson stream, on average 10 broken televisions per day (8 hours).

4. What is the fraction of time that the repair man has no work to do?
5. How many televisions are, on average, at his repair shop?
6. What is the mean throughput time (waiting time plus repair time) of a television?

>> Farm

In a dairy barn there are two water troughs (i.e. drinking places). From each trough only one cow can drink at the same time. When both troughs are occupied new arriving cows wait patiently for their turn. It takes an exponential time to drink with mean 3 minutes. Cows arrive at the water troughs according to a Poisson process with rate 20 cows per hour.

7. Determine the probability that there are i cows at the water troughs (waiting or drinking), $i = 0,1,2,\dots$
8. Determine the mean number of cows waiting at the troughs and the mean waiting time.
9. What is the fraction of cows finding both troughs occupied on arrival?

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10. How many troughs are needed such that at most 10% of the cows find all troughs occupied on arrival?

>> **Telephone System**

Suppose Bitways telephone system can keep a maximum of 5 calls on hold on any point in time. If a new call is made to the hotline when 5 calls are already in the queue, the new caller receives a busy signal. The calls arrive following a Poisson distribution with an average of 5 calls per minute. The service rate follows an exponential distribution with an average of 8 minutes necessary in order to finish a call with a customer. Currently there are two assistants available ready to answer the phone.

11. What's the probability that a call will be answered immediately?
12. What's the fraction of callers that receives a busy signal?
13. Calculate L , L_q , W and W_q .

>> **Miller Manufacturing**

Miller Manufacturing owns 10 identical machines used for the production of colored nylon. Machine breakdowns occur following a Poisson distribution with an average of 0.01 breakdowns occurring per operating hour per machine. The company loses \$100 each hour a machine is not working.

The company employs one technician to fix these machines whenever they break down. Service times to repair the machines are exponentially distributed with an average of 8 hours in order to repair one machine.

Management wants to analyze the impact of adding another service technician (service technicians are paid \$20 per hour).

14. What are the expected costs occurred if one technician is employed
15. What are the expected costs occurred if two technicians are employed