Stefan Helber and Florian Sahling

A Fix-and-Optimize Approach for the Multi-Level Capacitated Lot Sizing Problem

Summary by Christian Almeder

Outline

- **Problem definition (MLCLSP)**
- **Solution approaches in literature**
- **Fix-and-optimize heuristic**
- **Results**
- **Conclusions**

Problem Definition (MLCLSP)

- **Extension of the classical CLSP**
- **Multi-stage structure**

  Considering raw materials, subassemblies, final items and their relation to each other

  Difference to CLSP: additional term in the inventory balance constraint (internal demand)

$$Y_{k,t} = Y_{k,t-1} + Q_{k,t-z_k} - \sum_{i \in N_k} a_{k,i} \cdot Q - d_{k,t}$$

- **Production lead time z_k is used to consider delayed availability**
- **Overtime is used to relax the capacity constraint**
- **Problem is NP-hard -> no algorithm in polynomial time possible**

Problem Definition (MLCLSP)

- Important problem for the material requirements planning (MRP)
- Today's enterprise resource planning (ERP) systems often ignore capacities
- Fast solution methods are necessary (short-term planning)
- Solutions have to be disaggregated into a detailed production schedule

Solution Approaches in Literature

- **Mathematical programming approaches**

  Solution approaches based on solving the mixed-integer model (or parts of it)

  Valid inequalities

- **Lagrangean relaxation and decomposition**

  Relaxing the capacity and inventory balance constraints

  Decompose the problem into uncapacitated single-item problems

- **Local search and metaheuristics**

  Tabu Search, Genetic Algorithm, Ant Colony Optimization, …

Fix-and-Optimize Heuristic

- **Idea**

  MIP formulation hard to solve because of many binary variables

  Fix most of the binary variables (either to 0 or to 1)

  Optimize the remaining free binary variables.

  Iteratively change the fixed and free variables and optimize again

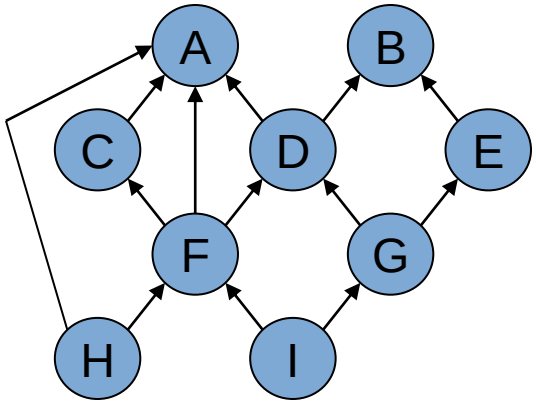
- **Necessary**

  Initial (starting) solution: Allow production for every item in every period (all γ_{kt}=1)

  Strategies to fix and free variables

Fix-and-Optimize Heuristic

- Product oriented decomposition
- Resource-oriented decomposition
- Process-oriented decomposition



Item	Periods						
	1	2	3	4	5	6	7
A (M1)	0	1	1	0	1	1	0
B (M2)	0	1	0	1	1	0	1
C (M3)	1	1	0	1	1	0	1
D (M1)	1	0	1	1	0	1	1
E (M3)	0	1	1	0	1	0	0
F (M2)	1	0	1	0	1	1	0
G (M2)	1	1	0	1	0	0	1
H (M1)	1	0	1	1	1	0	0
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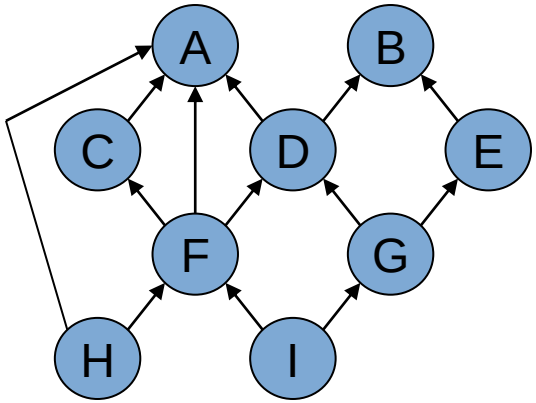

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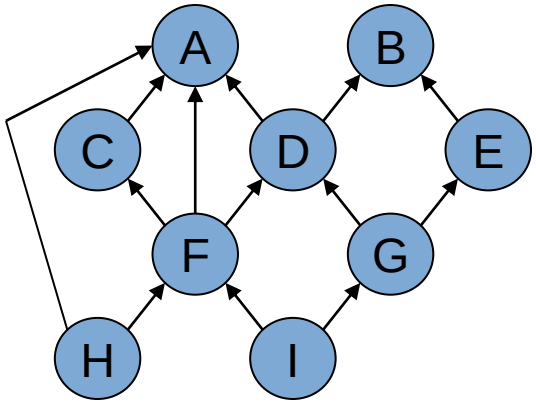

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I (M3)	1	1	0	0	1	0	1

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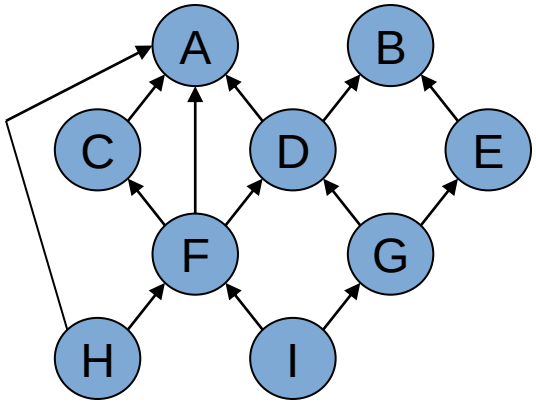

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G (M2)	1	1	0	1	0	0	1
H (M1)	1	0	?	?	?	?	0
I (M3)	1	1	0	0	1	0	1

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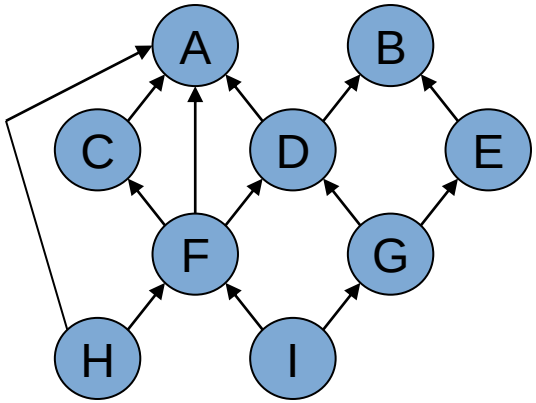

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	F (M2)	?	?	?	?	1	1	0
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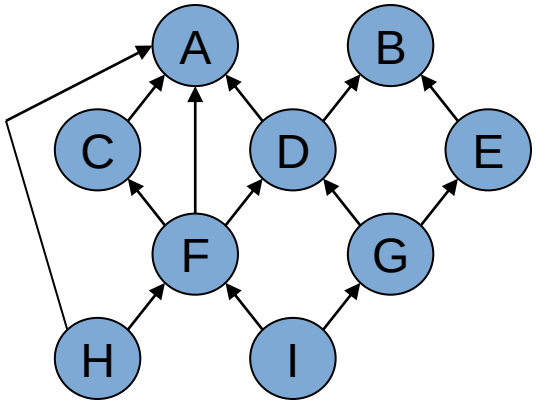

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E (M3)	0	1	1	0	1	0	0
F (M2)	1	0	1	0	?	?	?
G (M2)	1	1	0	1	0	0	1
H (M1)	1	0	1	1	1	0	0
I (M3)	1	1	0	0	1	0	1

Fix-and-Optimize Heuristic

- **Variant 1:**
  Product-oriented decomposition only
- **Variant 2:**
  Product-oriented decomposition
  Resource-oriented decomposition
- **Variant 3:**
  Product-oriented decomposition
  Process-oriented decomposition
- **Variant 4:**
  Product-oriented decomposition
  Resource-oriented decomposition
  Process-oriented decomposition

Results

- 5 classes of test instances (small, medium, large)
- Comparison with Tempelmeier and Derstroff (1996), Stadtler (2003), and truncated branch-and-bound (CPLEX)
- Tests with ($z_k = 1$) and without ($z_k = 0$) lead time

- Variant 4 gives the best solutions with the longest run time
- Variant 2 gives slightly better results than variant 3 in shorter time
- Variant 1 is for medium and large instances worse than Tempelmeier and Derstroff (1996)
- Proposed heuristic is faster than that by Stadtler (2003)

Conclusions

- **Method seems easy and fast**
- **Solution quality is good**

- **Criticism**
  - Method relays heavily on the performance of a commercial software product (CPLEX)
  - Solution quality reported by Stadtler (2003) are better than in this paper. Where comes the gap from?
  - Only comparison with two other methods (numerous papers on solution methods are cited)

What kind of information should be in a seminar work?

- **complete model formulation**
- **more information on the papers Tempelmeier and Derstroff (1996) and Stadtler (2003)**
- **Other papers dealing with a similar topic**

  Sahling, F., Buschkühl, L., Tempelmeier, H., Helber, S., Solving a multi-level capacitated lot sizing problem with multi-period setup carry-over via a fix-and-optimize heuristic. *Computers & Operations Research* 36, 2009, 2546-2553.
- **more detailed explanation of the algorithm**
- **deeper analysis of the results (comparison with other papers, e.g.: Stadtler, 2003; Pitakaso et al., 2006)**