Transportation Logistics Part I and II

Exercise 1

Let $G = (V, E)$ denote a graph consisting of vertices $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and arcs $A = \{(1, 2), (1, 3), (2, 4), (3, 2), (4, 3), (4, 5), (4, 6), (5, 3), (5, 7), (6, 8), (7, 4), (7, 6), (7, 8)\}$.

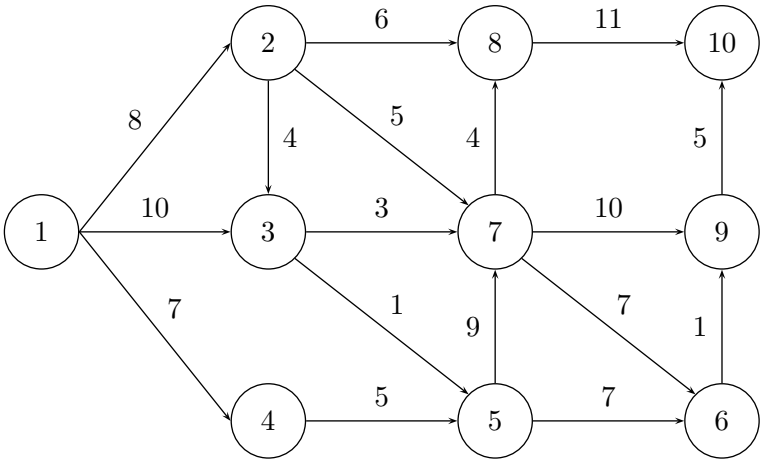
a) Draw graph G. Is G a tree, a digraph? Does G contain cycles? If yes, give an example.

b) Assume now that the arcs of G are in fact edges and that they have the following weights:

$c_{12} = 2, c_{13} = 3, c_{24} = 3, c_{32} = 2, c_{43} = 1, c_{45} = 3, c_{46} = 6,$
$c_{53} = 6, c_{57} = 7, c_{68} = 2, c_{74} = 4, c_{76} = 5, c_{78} = 3.$

Determine the minimum spanning tree using Kruskal's algorithm.

Exercise 2



a) Determine the shortest path between 1 and 10 with an algorithm of your choice.

b) In which case should the Bellman-Ford algorithm be employed?

c) ~~Assume now that the weights of the arcs correspond to arc capacities. Compute the maximum flow between vertex 1 and 10 (use the augmenting path algorithm) and identify a minimum cut.~~

Exercise 3

A weighted digraph with vertices $V = \{1, 2, 3, 4, 5\}$ is given by the following matrix (each entry denotes the weight of the arc connecting vertices i and j; ∞ indicates that the according arc does not exist.)

c_{ij}	1	2	3	4	5
1	0	∞	6	∞	∞
2	3	0	5	9	10
3	∞	4	0	7	2
4	∞	∞	6	0	9
5	∞	∞	∞	8	0

Draw the graph and use the Triple algorithm to find the shortest path between all vertices i and $j \in V$. In addition to the shortest distances, we are also interested in knowing the shortest path between each vertex pair.

~~Exercise 4~~

~~In Tirol, a new **waste incineration plant** shall be built. The inhabitants (in thousands) of the relevant municipalities are given in the following table:~~

~~Municipality~~	~~Inhabitants~~
~~Kitzbühel~~	~~100~~
~~Reutte~~	~~70~~
~~Kufstein~~	~~40~~
~~Innsbruck~~	~~90~~
~~Imst~~	~~50~~
~~Landeck~~	~~70~~

~~The travel times (in minutes) to and from each of the possible locations are assumed to be as follows:~~

	~~Kitzbühel~~	~~Reutte~~	~~Kufstein~~	~~Innsbruck~~	~~Imst~~	~~Landeck~~
~~Kitzbühel~~	~~0~~	~~3~~	~~5~~	~~12~~	~~7~~	~~7~~
~~Reutte~~	~~4~~	~~0~~	~~2~~	~~10~~	~~4~~	~~10~~
~~Kufstein~~	~~5~~	~~1~~	~~0~~	~~8~~	~~4~~	~~11~~
~~Innsbruck~~	~~8~~	~~9~~	~~2~~	~~0~~	~~6~~	~~13~~
~~Imst~~	~~5~~	~~3~~	~~7~~	~~6~~	~~0~~	~~7~~
~~Landeck~~	~~2~~	~~7~~	~~14~~	~~13~~	~~7~~	~~0~~

~~**a)** Determine the best location for the **waste incineration plant**.~~

~~**b)** Give an example for a practical decision problem where the determination of the In-Median is useful.~~

~~Exercise 5~~

~~Kärnten plans to build a new **fire department** for the municipalities Hermagor, Spittal an der Drau, Völkermarkt, Feldkirchen and Klagenfurt. The decision makers aim at~~