Transportation Logistics Part I and II

Exercise 1

Let $G = (V, E)$ denote a graph consisting of vertices $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and arcs $A = \{(1, 2), (1, 3), (2, 4), (3, 2), (4, 3), (4, 5), (4, 6), (5, 3), (5, 7), (6, 8), (7, 4), (7, 6), (7, 8)\}$.

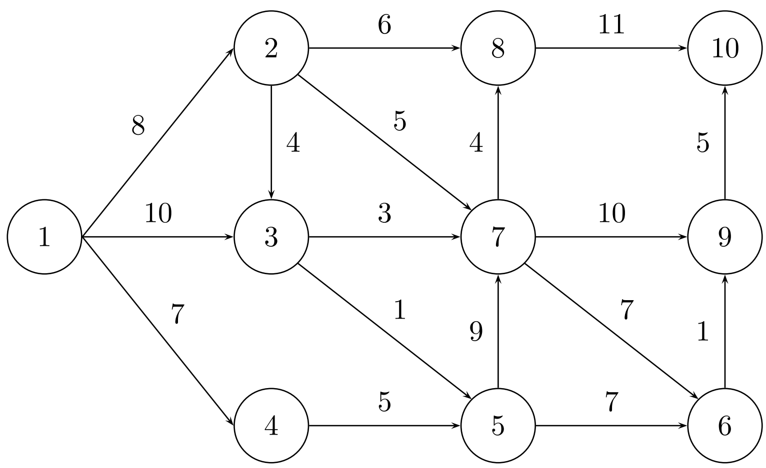
a) Draw graph G. Is G a tree, a digraph? Does G contain cycles? If yes, give an example.

b) Assume now that the arcs of G are in fact edges and that they have the following weights:

$c_{12} = 2, c_{13} = 3, c_{24} = 3, c_{32} = 2, c_{43} = 1, c_{45} = 3, c_{46} = 6,$
$c_{53} = 6, c_{57} = 7, c_{68} = 2, c_{74} = 4, c_{76} = 5, c_{78} = 3.$

Determine the minimum spanning tree using Kruskal's algorithm.

Exercise 2



a) Determine the shortest path between 1 and 10 with an algorithm of your choice.

b) In which case should the Bellman-Ford algorithm be employed?

c) ~~Assume now that the weights of the arcs correspond to arc capacities. Compute the maximum flow between vertex 1 and 10 (use the augmenting path algorithm) and identify a minimum cut.~~

Exercise 3

A weighted digraph with vertices $V = \{1, 2, 3, 4, 5\}$ is given by the following matrix (each entry denotes the weight of the arc connecting vertices i and j; ∞ indicates that the according arc does not exist.)

c_{ij}	1	2	3	4	5
1	0	∞	6	∞	∞
2	3	0	5	9	10
3	∞	4	0	7	2
4	∞	∞	6	0	9
5	∞	∞	∞	8	0

Draw the graph and use the Triple algorithm to find the shortest path between all vertices i and $j \in V$. In addition to the shortest distances, we are also interested in knowing the shortest path between each vertex pair.

Exercise 4

In Tirol, a new **waste incineration plant** shall be built. The inhabitants (in thousands) of the relevant municipalities are given in the following table:

Municipality	Inhabitants
Kitzbühel	100
Reutte	70
Kufstein	40
Innsbruck	90
Imst	50
Landeck	70

The travel times (in minutes) to and from each of the possible locations are assumed to be as follows:

	Kitzbühel	Reutte	Kufstein	Innsbruck	Imst	Landeck
Kitzbühel	0	3	5	12	7	7
Reutte	4	0	2	10	4	10
Kufstein	5	1	0	8	4	11
Innsbruck	8	9	2	0	6	13
Imst	5	3	7	6	0	7
Landeck	2	7	14	13	7	0

a) Determine the best location for the **waste incineration plant**.

b) Give an example for a practical decision problem where the determination of the In-Median is useful.

Exercise 5

Kärnten plans to build a new **fire department** for the municipalities Hermagor, Spittal an der Drau, Völkermarkt, Feldkirchen and Klagenfurt. The decision makers aim at determining the best location for the **fire department**. The shortest travel times (in minutes) between all municipalities have already been computed:

	Hermagor	Spittal	Völkermarkt	Feldkirchen	Klagenfurt
Hermagor	0	60	80	70	60
Spittal	50	0	70	50	50
Völkermarkt	70	60	0	50	30
Feldkirchen	60	60	40	0	30
Klagenfurt	50	50	20	40	0

The number of inhabitants per municipality are as follows:

Municipality	Inhabitants
Hermagor	8.000
Spittal	12.000
Völkermarkt	13.000
Feldkirchen	10.000
Klagenfurt	90.000

a) Should the In-Median or the Out-Median problem be solved?

b) Solve the Median problem for the given data.

Exercise 6

For several years, a French wholesale chain has been exporting red wine to five retail stores in Austria. Until now, the products were sent by air cargo from Bordeaux to Vienna. From there, the bottles were transported directly to the customers. Since the demand for this type of red wine has been constantly growing the headquarters in Bordeaux decided to expand their commercial operations in Austria. In a first step, several distributing warehouses shall be built. Four potential locations have been determined: Klagenfurt, Linz, Baden and Innsbruck. Based on historical data, the future demand at the five retail stores can be estimated: 10, 25, 40, 18, 8. Estimates for the transportation costs from each of the potential sites to the retail stores are also available. These data are given in the following table:

	1	2	3	4	5
Klagenfurt	14	12	10	20	10
Linz	20	10	9	14	16
Baden	15	9	22	15	8
Innsbruck	8	20	10	22	24

Fixed setup costs occur for the distributing warehouses. Building a warehouse in Klagenfurt would cost 18 monetary units; realizing a warehouse in Linz costs 23 monetary units; in Baden it would cost 3 monetary units and for Innsbruck 12 monetary units are assumed.

a) What kind of problem do we have to solve? Based on the available data, formulate the problem in terms of a linear program.

b) How many continuous variables and how many binary variables are needed?

c) Solve the problem using the ADD heuristic and determine the total costs of your solution.

Exercise 7

A wood-processing company aims at constructing one or more warehouses in order to serve 5 customers with wood products. Four different possible warehouse locations are available. The travel times (in minutes) from each of the potential sites to each of the five customers and back are given in the following table:

	I	II	III	IV	V
A	140	120	100	200	100
B	150	90	220	150	80
C	200	100	90	140	160
D	80	200	100	220	240

Which warehouses should be realized if one hour of truck driver labor costs 300 monetary units? The fixed setup costs for each of the warehouses are $f_A = 1000$, $f_B = 800$, $f_C = 1200$, $f_D = 900$.

a) Solve the problem using the ADD heuristic.

b) Solve the problem using the DROP heuristic.

c) Compare the objective function values obtained from the two methods. Are the obtained solutions optimal (argue why or why not)?

Exercise 8

Assume the following transportation problem:

c_{ij}	1	2	3	4	s_i
1	10	5	6	7	25
2	8	2	7	6	25
3	9	3	4	8	50
d_j	15	20	30	35	

a) Compute a basic feasible solution with a method of your choice. Give reasons for your choice.

b) Compute the optimal solution using the MODI method.

Exercise 9

A company in the construction business serves 4 wholesalers (j) from 4 warehouses (i). Transportation costs, demand and supply are given in the following table:

c_{ij}	1	2	3	4	s_i
1	10	12	23	14	15
2	12	8	16	15	15
3	12	11	9	17	20
4	4	6	8	8	50
d_i	15	20	30	35	100

The currently realized solution is given in the following table:

i,j	1	2	3	4	s_i
1				15	15
2	15				15
3		20			20
4	0	0	30	20	50
d_i	15	20	30	35	100

a) Is the provided solution optimal? If not, determine the optimal solution.

b) By how many units may the capacity of warehouse 1 and the demand of customer 1 be changed such that the computed basic solution remains optimal?

c) Does the solution remain optimal if the transportation costs from warehouse 3 to customer 1 are reduced by 2 monetary units?

Exercise 10

A production company disposes of 3 production sites W_i, $i = 1, 2, 3$, for a given product. In W_1, 4 units are produced, in W_2 21 and in W_3 15. There are three customer locations K_j, $j = 1, 2, 3$ with a demand of 8, 15, and 10, respectively. A special supply agreement states that at least 5 units have to be delivered from W_2 to K_2 and 3 units from W_3 to K_3. The transportation costs are the following:

$$C = \begin{matrix} 3 & 9 & 9 \\ 2 & 11 & 14 \\ 8 & 10 & 11 \end{matrix}$$

a) Formulate the problem situation in terms of a LP.

b) Reduce the problem to the transportation problem.

c) Determine a feasible solution using the column minima method.

d) Solve the problem to optimality. Use the solution of the column minima method as starting solution. (no cost reduction)

e) Does the obtained solution remain optimal if the transportation costs from W_3 to K_1 are increased by 2?

f) How does the optimal objective value change if the supply of W_1 and the demand at customer 3 are reduced by 2 units?

Exercise 11

An organic farmer association runs 3 warehouses to distribute their vegetables to 4 restaurants. In each of the warehouses 250, 300 and 250kg of vegetables may be stored respectively. Restaurant R1 has ordered 150 kg, R2, 50 kg and R3 350 kg of vegetables. R4 has not ordered yet. The association estimates that in addition to the already ordered quantities, R1 could sell 150 kg, R2, 150 kg and R4 250 kg (no additional quantities at R3). The transportation costs are as follows (R3 cannot be served by K3):

cij	R1	R2	R3	R4
K1	16	13	22	17
K2	19	20	23	15
K3	14	13	-	19

Assume that the already ordered demand has to be satisfied and that the entire quantity in stock shall be sold. A transportation plan that minimizes the total transportation costs is required.

a) Formulate the problem as a transportation problem. Introduce a dummy warehouse and divide each R_i into two customers (one for the already ordered demand and one for the additional demand).

b) Solve the problem using Vogel's approximation method and the MODI method. Infeasible connections should be associated with prohibitively high transportation costs.

Exercise 12

A furniture company is planning to establish at most 4 warehouses to serve 5 furniture stores with wooden tables. Four possible locations were determined. The travel times (in minutes) from the potential warehouses to the furniture stores and back are as follows:

	I	II	III	IV	V
A	140	120	100	200	100
B	200	100	90	140	160
C	150	90	220	150	80
D	80	200	100	220	240

Which warehouses shall be built if one hour of labor costs 300 monetary units. The fixed setup costs per location are $f_A = 1000$, $f_B = 1200$, $f_C = 800$, $f_D = 900$.

The demands of the five furniture stores are the following:

I: 1000 units

II: 2500 units

III: 4000 units

IV: 1800 units

V: 600 units

The warehouses have the following capacities:
A: 1500 units
B: 1700 units
C: 4500 units
D: 4200 units

Solve the problem using the DROP heuristic.

Exercise 13

A maintenance company has to complete 5 different tasks at company owned locations. Six technicians are available. Each of them will need a different amount of time for each of the tasks:

i, j	Task 1	Task 2	Task 3	Task 4	Task 5
Tech 1	10	15	30	20	30
Tech 2	15	30	25	25	35
Tech 3	25	20	20	10	25
Tech 4	15	25	25	20	20
Tech 5	25	15	15	20	25
Tech 6	25	30	20	25	25

Find the optimal technician to task assignment such that the total duration needed to accomplish all tasks is minimized.

Exercise 14

An Austrian wine producer is planning to send their four winemakers on a training week to different well known wine making regions. Each of the winemakers should visit a different region. Four regions were selected: Bordeaux, Adelaide, Porto and Tuscany. The different wine makers have different preferences for the different regions:

i,j	Bordeau	Adelaide	Porto	Tuscany
W1	10	5	4	3
W2	5	4	7	11
W3	5	5	4	11
W4	6	6	8	5

Find the optimal solution maximizing the total preference score across all wine makers. (Hint: the maximization problem can be transformed into a minimization problem: take the maximum preference score and replace the preference value in each cell with the difference to the maximum. Now the deviation from the maximum score can be minimized.)