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Solutions to the request reassignment problem in collaborative carrier networks

Susanne Berger, Christian Bierwirth *

School of Economics and Business, Martin-Luther-University Halle-Wittenberg, Germany

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ABSTRACT

The paper considers an arrangement for exchanging transportation requests to facilitate collaboration among independent carriers. The goal is to maximize the total profit without decreasing the individual profit of the carriers. Two solution approaches are developed for this problem involving decentralized control and auction based exchange mechanisms. The results are compared with those obtained without collaboration and by a centralized control. They indicate that horizontal collaboration pays off even in highly competitive environments.

1. Introduction

The idea of forming networks of collaborating carriers is well known in practice. In the European less-than-truckload market, six of the top ten carrier organizations are actually networks of collaborating small- and medium-sized companies (Klaus, 2003). While large carriers can generate economies of scope on their own, smaller companies usually need to search for load complementing the current operations in order to balance their capacities. For these firms collaboration has become a strategic necessity to gain efficiency in face of decreasing profit margins.

In this paper we consider a network of collaborating freight carrier companies that provide an equivalent transport service in their regional areas. Every day the carriers receive new transportation requests either from shippers, typically manufacturers and retailers, or large carrier organizations as sub-contracts. On this basis the carriers plan their daily operations. As a result it can turn out that some requests cannot be efficiently integrated into the route of a carrier. In this situation, a post-market and optimization-based collaboration can yield a reassignment of the requests which improves both, the overall efficiency of the network and the individual profit of each single carrier.

We propose a framework for a post-market based optimization, that might be implemented as an internet based electronic platform. The goal of the framework is to maximize the overall profit of the network with as few as possible information transfer, because carriers disclose customer information unwillingly. The framework relies on true operation cost by integrating exact algorithms for transportation planning. In the paper we concentrate on a pickup and delivery service, where the shipments take only a fraction of the vehicle capacity, like in courier services. This simplification is often made in the literature, when transport capacity goes beyond the scope of research, compare, e.g. Gendreau and Potvin (1998), Mitrovic-Minic et al. (2004) and Shen et al. (1995).

However, the particular selection of which problem to solve within the framework is not central to our approach. Here, it leads to an uncapacitated pickup and delivery problem, also known as the traveling salesman problem with precedence

* Corresponding author.
E-mail addresses: susanne.berger@wiwi.uni-halle.de (S. Berger), christian.bierwirth@wiwi.uni-halle.de (C. Bierwirth).

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constraints (TSPPD). More general routing problems considering multiple vehicles, transport capacities, etc., can be integrated in the framework as well. E.g. if the load of shipments is significant for a service, a capacitated routing problem has to be solved instead within the framework.

The approach allows us to assess a carrier collaboration strategy on a numerical basis. We distinguish between three different strategies in which (a) the carriers do not collaborate, (b) the carriers collaborate through a decentralized planning approach, and (c) the carriers collaborate through a central planning approach, where full information is needed. Regarding strategy (b) we derive two algorithms from our framework where a Vickrey Auction and a combinatorial auction are implemented. The functionality of the framework can be used for theoretical investigations on the profitableness of carrier cooperation under particular conditions. In this paper we investigate the impact of competition among the carriers by varying the geographical separation of their customer areas in three steps (adjacent, overlapping, identical).

The paper is organized as follows. The next section briefly reviews the literature on collaboration in transportation markets. Section 3 provides a model for accounting and reassigning transportation requests in the collaborative carrier network. Procedures for reassigning requests must be confidential and effective in order to create incentives for carriers to participate in the collaboration. A general procedure that meets these requirements is presented in Section 4. Section 5 reports the achieved computational results.

2. Literature review

In transportation markets collaboration is basically established in two directions: Shippers collaborate to increase the competition among carriers and carriers collaborate in order to balance their transport capacities. The first type of collaboration has received only little attention in the literature. Recently, the Shipper Collaboration Problem has been formulated in Ergun et al. (2007). In this work, shippers try to identify sets of lanes, with little asset repositioning, and submit it to the carriers as a bundle to get more favorable rates.

Collaboration between carriers aims at an optimal usage of the available transport capacities. In a network of collaborating carriers an assignment of a set of shipper requests to carriers has to be found which minimizes the total operation cost. Various aspects of collaborative carrier networks have been investigated in the literature, including sub-contracting decisions, negotiation and auction mechanisms, profit sharing and communication platform design.

In the approaches of Sandholm (1993) and Fischer et al. (1996) a network of independent carriers is considered. The carriers are represented by agents that communicate and act on a market platform. Sandholm (1993) considers a market where carriers offer requests which are sub-contracted by other carriers as a result of a bidding process. The market platform proposed by Fischer et al. (1996) allows carriers to exchange requests and information about free transport capacities. These actions are coordinated through bilateral negotiations between the carriers based on the local planning situations.

In the approach of Gomber et al. (1999), a large carrier organization is divided into regional profit centers. Incoming transportation requests of shippers are assigned to one of the profit centers using a Vickrey Auction. This type of auction ensures that true values are revealed for each request. In this work and in the one of Sandholm (1993), more complex contracts of exchanging bundles of requests are also suggested. To arrange a fair splitting of the gained profit, the mentioned approaches use more or less embellished clearing systems.

The recomposition problem for exchanging requests between the collaborating carriers is described by Schönberger (2005). A heuristic two-step approach is applied where each carrier selects bundles of requests leading to a maximum profit contribution. If several carriers compete for a request, a mediator brings about the assignment decision. Requests not assigned to one of the carriers are sub-contracted to external carriers. The incurred costs are shared between the collaborating carriers. The approach suffers from the assumption of information transparency and the lack of incentives to collaborate. An interesting remedy is proposed by Krajewska and Kopfer (2006). They introduce a three-step procedure which creates win–win situations for the carriers. In the pre-processing step the carriers announce self-fulfillment cost for their own requests. Afterwards, they bid on bundles of offered requests by estimating their individual fulfillment cost. In the profit optimization step the Winner Determination Problem as formulated by de Vries and Vohra (2003) is solved for the resulting combinatorial auction. In the final step, the saved cost is transferred into a profit gain which is split up among the concerned carriers according to a collaboration-advantage-index.

In a further stream of research the use of auctions is considered in the literature for the procurement of transportation services. Basically, this is a complex form of outsourcing, where the shippers offer transportation charges or lanes to carriers on a regular basis. Decisions on such business relations are made at a tactical level without involving collaboration. However, these approaches are similar to the collaborative approaches discussed above because they aim at increasing the efficiency of the participating actors in the transportation market as well. From the shippers' view significant cost reductions can be obtained if the required transportation services are bought through a request for proposal process. At the same time carriers can benefit from economies of scope in bidding on lane contracts which can be fulfilled by a minimum amount of empty load travels (Caplice and Sheffi, 2003). Particular advantage arises from initiating combinatorial auctions where the carriers can bid on bundles of lanes. Sheffi (2004) reports that combinatorial auctions are widely accepted for the procurement of transportation services in the US truckload transportation market. Elmaghraby and Keskinocak (2003) discuss the use of a single round combinatorial auction at Home Depot Inc. for procuring truckload service. A multiple round combinatorial auction for Sears Inc. is reported by Ledyard et al. (2002). After each round provisional winners are announced by the auctioneer and the carriers can submit new bids, taking into account the received information. Such multiple round auctions are of interest if little is known about the true cost savings that can be obtained from bundled offers.

In order to identify optimal bids for bundles of lanes, carriers face the Bidder Optimization Problem introduced by Song and Regan (2002). In Song and Regan (2005) the 0–1 enumeration method of Balas is used to solve the problem with and without pre-existing commitments made by the carriers. In this approach the cost for empty travels is minimized. Carriers calculate it by computing optimal truck routes for the lanes of a bundle. Since these empty travels do not necessarily occur in the implemented routing plan, the operation costs are roughly approximated. In the approach of Lee et al. (2007) routes are constructed by optimally trading off repositioning costs of vehicles and the rewards associated with servicing lanes. This model comprises a simultaneous generation of vehicle routes and lane selection. Column generation and Lagrangian techniques are proposed for solving the resulting optimization problem.

Current research has pointed out the impact of individual operation cost on the reallocation of requests. Nevertheless, existing concepts predominantly rely on estimates of the operation cost which are provided by heuristics for solving the underlying routing problem. Therefore, these studies do not allow accessing the utility of collaboration on a quantitative basis. Furthermore, the impact of providing information on the gain of collaboration is hardly discussed in the literature. In the absence of full information transparency, collaborative planning problems cannot be solved centrally. In order to fill these gaps, a framework for decentralized decision making in a carrier network is proposed in the paper. The framework involves exact algorithms for transportation planning in order to determine the true cost of decentralization against a central planning approach. It enables simulation and quantitative analysis of the network transactions and thus can provide theoretical insight into the potentials of particular network configurations.

3. The Collaborative Carrier Routing Problem

This section defines the problem of reassigning transportation requests in a Collaborative Carrier Network (CCN) such that the total profit is maximized. The problem is referred to as the Collaborative Carrier Routing Problem (CCRP).

3.1. Framework and notation

In the CCRP we refer to a set of carrier companies that offer an equivalent freight transportation service to shippers. It is assumed that the carriers operate from individual depots and use a homogeneous fleet. In our model, shippers request capacity for origin-destination pairs which are given in terms of pickup and delivery locations. For simplicity we focus on a service, where load capacity can be ignored because the shipments take only a very small fraction of the vehicle capacity.

A CCN consists of m independent carrier companies of the described type. They form the member set M of the network. Let N_i denote the set of requests contracted by carrier $i \in M$. Furthermore, let N denote the set of all requests, contracted by the members of the CCN. We assume that all requests have to be served within the same period and that the carriers perform their operations planning on a periodic basis (e.g. daily).

The collaboration framework consists of a mechanism to exchange requests between carriers and a corresponding cash flow model for the possible network transactions. Moreover, the framework includes a calculation scheme for the determination of revenue, cost and profit. For a formal description of these components the following notation is used.

M	the member set of the carrier network, $M = \{1, 2, \ldots, m\}$
N	set of all requests to be served in one period, $N = \{1, 2, \ldots, n\}$
N_i	set of requests contracted by carrier $i \in M$ ($N = \cup_{i \in M} N_i$)
r_j	revenue of request $j \in N$ paid by a shipper
c_{ij}	marginal cost of carrier $i \in M$ to serve request $j \in N$
p_{ij}	marginal profit of request $j \in N$ for carrier $i \in M$
d_j	direct traveling distance of request $j \in N$
$\alpha_1,\ \alpha_2$	base rate and distance dependent transportation rate per kilometer
l_{ij}	marginal tour length of request $j \in N$ for carrier $i \in M$
$\beta_1,\ \beta_2$	stopping cost per request and traveling cost per kilometer
$L(N')$	minimum tour length needed to serve a set of requests N'
R_i	total revenue of carrier $i \in M$ gained for serving N_i
C_i	total cost of carrier $i \in M$ incurred from serving N_i
P_i^0	total profit of carrier $i \in M$ obtained without collaboration
P_i	total profit of carrier $i \in M$ obtained for serving request set N_i
P	period profit of the network
u_j	takeover price for request $j \in N$ in a reverse auction
v_j	compensation price for request $j \in N$ in a forward auction
x_{ij}	binary variable, set to 1 if carrier $i \in M$ serves request $j \in N$
ΔP	profit gain of the network, achieved by a reassignment of requests

3.2. Profit determination without collaboration

In our model, the carriers calculate freight charges using a two-stage pricing system. The charge or revenue r_j of a request is composed of a basic transportation rate α_1 and a distance dependent transportation rate α_2

$$r_j = \alpha_1 + d_j \cdot \alpha_2, \tag{1}$$

where d_j denotes the direct distance between the pickup and the delivery location. The transportation volume does not affect the charge because vehicle and load capacities are ignored. From the customer's point of view, the charge to be paid represents a reasonable approximation of the service's value. The total revenue of carrier i is determined by $R_i = \sum_{j \in N_i} r_j$.

In transportation markets, the paid charges typically hide the true cost of service creation. Let c_{ij} denote the self-fulfillment cost as the marginal cost of carrier $i \in M$ to serve request $j \in N_i$. The direct cost of the request consists of stopping cost β_1 for waiting, loading and unloading times at the pickup and delivery locations. Furthermore, a traveling cost rate per kilometer, denoted as β_2, is involved in the marginal cost. This leads to

$$c_{ij} = \beta_1 + l_{ij} \cdot \beta_2, \tag{2}$$

where l_{ij} represents the marginal tour length of request j. It is defined by the additional traveling distance for carrier i to serve request j. The marginal tour length is computed by the difference of the tour lengths required to serve the request set N_i including and excluding request j

$$l_{ij} = L(N_i) - L(N_i \setminus \{j\}). \tag{3}$$

Here, function $L(\cdot)$ returns the tour length which we calculate by solving the underlying routing problem to optimality. As already mentioned, we concentrate on the TSPPD in this paper. However, Eqs. (2) and (3) reveal that the cost calculation is virtually independent of the involved routing problem. Regardless of the involved type of routing problem, the total cost of carrier i is given by $C_i = \sum_{j \in N_i} \beta_1 + L(N_i) \cdot \beta_2$. This function is generally convex and non-additive because $L(N_i) \neq \sum_{j \in N_i} l_{ij}$. Economies of scope are reflected by the inclusion of the marginal tour length. The marginal profit of a request is given by $p_{ij} = r_j - c_{ij}$. The carrier's profit per period amounts to $P_i^0 = R_i - C_i$. Without collaboration the profit of the CCN yields $P = \sum_{i \in M} P_i^0$.

Table 1 gives an example for the profit calculation. The coordinates of the depot of Carrier 1 as well as the pickup and delivery locations of three transportation requests are randomly taken from a large TSP instance. The transportation distance d_j corresponds to the rounded Euclidean distance. Using price rates $\alpha_1 = 20$ and $\alpha_2 = 2$, the revenue is $R_1 = 180$. The minimum tour length is $L(N_1) = 150$. With stopping cost $\beta_1 = 10$ and traveling cost per kilometer of $\beta_2 = 1$, the carrier's total cost is $C_1 = 180$. This means he makes no profit $(P_1 = 0)$, although two requests generate significant marginal profit.

3.3. Sub-contraction of requests

In a CCN, the carriers $i \in M$ can decide to self-fulfill their contracted requests. Alternatively, they can attempt to sub-contract a subset of the requests to other members of the network. Sub-contracting requests is considered to be attractive in case of low marginal profit. It can be performed in different ways.

Assume that carrier i has contracted transportation request j at charge r_j from a shipper. Assume furthermore that another carrier, say k, is able to serve request j at marginal cost $c_{kj} < c_{ij}$. In this situation carrier i can either forward the request to k and pay a takeover price u_j or sell the request and receive a compensation price v_j from k. In the former case, carrier i receives the charge from the shipper and in the latter carrier k. Takeover prices are typically negotiated in a reverse auction while compensation prices are negotiated in a forward auction (Turban et al., 2005). The different cash flow models of both transactions are shown in Fig. 1.

In a reverse auction, the maximum willingness to pay of carrier i, called the reservation price, is bounded by his marginal cost to serve request j, i.e. $u_j \leqslant c_{ij}$. Accordingly, in a forward auction, a floor price expressing the minimum compensation price accepted by carrier i is bounded by the marginal profit of request j, i.e. $v_j \geqslant p_{ij}$. Sub-contracting by takeover prices aims at total cost minimization, while sub-contracting by compensation prices aims at total profit maximization. From the

Table 1
Individual profit calculation for Carrier 1.

Requests j	Revenue		Cost		Profit p_{1j}
	d_j	r_j	l_{1j}	c_{1j}	
1	10	40	38	48	-8
2	32	84	17	27	57
3	18	56	37	47	9
	$\sum d_j$	R_1	$L(N_1)$	C_1	P_1
N_1	60	180	150	180	0

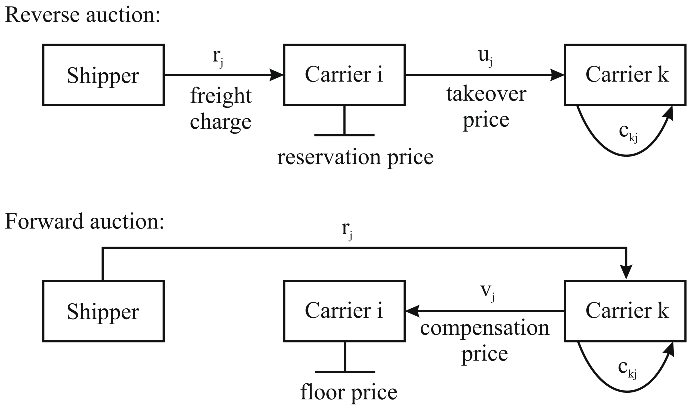

Fig. 1. Cash flows for sub-contracting by takeover prices and compensation prices.

network's point of view, takeover prices and compensation prices serve only as internal transfer prices. Therefore, both cash flow models support the allocation of network resources in a compared manner, with the difference that the responsibility for a request is completely put on the conducting carrier in a forward auction. If we view a collaborative network as a closed virtual enterprise with a common external appearance, the question of responsibility is of secondary concern. This allows us to concentrate on network profit maximization using forward auctions for the reassignment of transportation requests to carriers.

Let x_{ij} be a binary variable indicating whether $j \in N$ is served by $i \in M$ ($x_{ij} = 1$) or not ($x_{ij} = 0$). Extending the profit evaluation scheme, carrier i determines the marginal profit for requests $j \in N$ as follows:

$$p_{ij} = \begin{cases} (r_j - c_{ij}) \cdot x_{ij} + v_j \cdot (1 - x_{ij}), & \text{if } j \in N_i, \\ (r_j - c_{ij} - v_j) \cdot x_{ij}, & \text{otherwise}. \end{cases} \tag{4}$$

Here, r_j and c_{ij} are computed according to (1) and (2). In case of self-fulfillment ($j \in N_i$ and $x_{ij} = 1$) the formula corresponds to the ordinary profit computation. In case that the carrier has contracted j but decides to sub-contract it again ($j \in N_i$ and $x_{ij} = 0$), the profit is given by the compensation price v_j paid by the sub-contractor. Finally, if the carrier appears as sub-contractor ($j \notin N_i$ and $x_{ij} = 1$), the marginal profit of the request results from the revenue minus the marginal cost and the paid compensation price.

3.4. The CCN optimization problem

The optimization problem of the CCN is to find a reassignment of requests to carriers such that the total profit of the CCN is maximized, assuming the carriers provide an optimal routing of their vehicles.

In what follows, we assume carriers to be willing to sell a customer request, provided they obtain an acceptable compensation price. A transfer of request j from carrier i to carrier k is represented in the framework by j leaving N_i and entering N_k. Hence, the goal of the CCN is to reassign requests to carriers with respect to the paid compensation prices such that the period profit is maximized. Using the binary decision variable x_{ij}, the CCRP is formulated as follows:

$$\max \quad P = \sum_{j \in N} r_j - \sum_{i \in M} C_i \tag{5}$$

$$\text{s.t.} \quad 1 = \sum_{i \in M} x_{ij} \quad (\forall j \in N) \tag{6}$$

$$C_i = \sum_{j \in N} \beta_1 x_{ij} + L(\{j \in N | x_{ij} = 1\})\beta_2 \quad (\forall i \in M) \tag{7}$$

$$P_i^0 \leqslant \sum_{j \in N} r_j x_{ij} + \sum_{j \in N_i} v_j (1 - x_{ij}) - \sum_{j \in N \setminus N_i} v_j x_{ij} - C_i \quad (\forall i \in M) \tag{8}$$

$$x_{ij} \in \{0, 1\} \quad \text{and} \quad v_j \in \mathbb{R} \quad (\forall i \in M, \forall j \in N) \tag{9}$$

The objective function (5) maximizes the period profit of the CCN as the total revenue of all requests minus the total cost of all carriers. Since the total revenue is invariant, we can alternatively minimize the total cost alone. Constraints (6) ensure that every request is assigned to exactly one carrier. The transportation costs of the carriers are computed in constraints (7) with respect to a request-to-carrier assignment. Note that the constraints of the involved TSPPD also nest inside (7), because the determination of $L(\cdot)$ requires to solve the addressed problem instance. The initially given assignment of request to carriers is represented in the model by the sets N_i. Constraints (8) ensure that the profit of the carriers does not deteriorate if requests are reassigned to other carriers. In a pre-processing, the initial profit P_i^0 that the carriers can achieve without collaboration, is calculated. The new profit is composed of the revenues and compensation prices obtained by a carrier minus the compensation prices paid and the transportation cost incurred from the reassignment.

We have formulated the above MIP model to provide a straightforward mathematical description of the general CCRP. It can be seen that the model is non-linear due to constraint (8). This non-linearity, however, is avoided by replacing variable v_j by v_j^{ik}, denoting the price paid by carrier i to carrier k for taking over transportation request j. Now, constraint (8) changes into the following linear constraints:

$$P_i^0 \leqslant \sum_{j\in N} r_j x_{ij} + \sum_{j\in N_i} \sum_{k\in M\setminus\{i\}} v_j^{ik} - \sum_{j\in N\setminus N_i} \sum_{k\in M\setminus\{i\}} v_j^{ki} - C_i \quad (\forall i \in M) \tag{10}$$

$$v_j^{ik} \leqslant B \cdot x_{kj} \quad (\forall i \in M, k \in M \setminus \{i\}, j \in N_i) \tag{11}$$

Here, constraint (11) ensures the coupling of the compensation prices and the decision variables, where B denotes a big number. Due to the complexity of the involved routing problem, even small instances of the general CCRP cannot be solved by standard MIP solvers. As pointed out below, such instances can be solved by considering the general problem as a multi-depot variant of the specific type of routing problem instead.

An important observation taken from the CCRP model is that the compensation prices are not fixed by an allocation of requests to carriers. Although the compensation prices are decision variables in the model, they reflect only internal transfer prices in the network and thus do not appear in the objective function. Typically, there are many feasible constellations of compensation prices. They can also take negative values if the marginal profit of a request is negative for the acquiring carrier. The model merely ensures the existence of compensation prices which satisfy the requirements of a win-win situation. Hence, the question of how to share the savings gained from the collaboration is basically independent from the CCRP. Nevertheless, the disposition of carriers to participate in a collaboration strongly depends on the way the profit is shared among the participants. Therefore, profit sharing methods like the collaboration advantage index and the Shapley value have received considerable attention in the literature, see Krajewska and Kopfer (2006) and Shapley (1953). Since we do not primarily address the profit sharing problem, a simple uniform sharing of the collaboration gain is applied subsequent to the optimization phase.

4. Procedures for the reassignment of requests

Presupposed the demand of transport capacity and the distribution of requests are centrally known, the CCRP can be solved as a multi-depot pickup and delivery problem (Lu and Dessouky, 2004). We consider an uncapacitated variant of this problem, referred to as the multi-depot traveling salesman problem with pickup and deliveries (MDTSPPD). In this problem, a set N of transportation requests has to be assigned to a set M of carriers (or depots) such that the total cost is minimized.

In the absence of information transparency, the CCRP cannot be solved like a multi-depot problem. Instead of assigning requests to carriers centrally, requests have to be reassigned, which requires the agreement of the concerned carriers. Hence, a transfer procedure for reassigning customer requests among autonomous acting agents is needed. Such a procedure must be confidential and effective in terms of substantial incentives for the carriers to participate in the process.

4.1. Idea

Starting from the initial distribution of requests $N = N_1 \cup N_2 \cup \cdots \cup N_m$, we search for a reassignment of requests to carriers that increases the overall network profit. This procedure is described in five steps.

1. *Forming a request candidate set*: Every carrier $i \in M$ chooses request $j_i \in N_i$ with the lowest marginal profit as a candidate for a possible reassignment. The value of p_{ij_i} is viewed by the carrier as a floor price for request j_i. The set of candidate requests in the network is $S = \{j_1, j_2, \ldots, j_m\}$.
2. *Composition of bundles from the candidate set*: A number of bundles $S_k \subseteq S (k = 1, \ldots, s)$ is selected for the reassignment process. In the simplest case there is only a single bundle containing a single request. In the other extreme all subsets of S define bundles.
3. *Determination of marginal profits*: Every carrier i determines the marginal profit p_{ik} for each bundle S_k of requests, without taking into account the corresponding compensation prices.
4. *Assignment of bundles to carriers*: The bundles are tentatively assigned to carriers such that the sum of the related marginal profits, computed in Step 3, is maximized. If a request is contained in multiple bundles, only one of these bundles is assigned to a carrier.
5. *Profit sharing*: If the period profit of the CCN has increased by the reassignment, the profit gain is split up among the concerned carriers. Otherwise, the attempted reassignment of requests has failed.

These steps are iteratively repeated until no further improvement is possible. Note that Steps 1 and 3 are autonomously performed by the carriers whereas Steps 2, 4, and 5 are centrally performed.

The above procedure can be viewed as the structure of an electronic exchange market, where the carriers are both, buyers and sellers. The composed bundles of requests are the negotiated items. The market requires coordination, for instance executed through an auction mechanism. Generally auctions are a form of multilateral negotiations where participants interact on the basis of bids (McAfee and McMillan, 1987). The marginal profit which is observed by the carriers for each offered bundle can take the role of bids in the auction.

In the following, we derive two algorithms for the CCRP from the above procedure. The first algorithm is most simple and considers only a single bundle containing a single request in Step 2. It is supposed that the carriers do not know the candidate set in advance and that the requests from the set are reassigned one by one. Hence, the carriers must consider each auctioned request independent of the succeeding ones. In the second algorithm the candidate set is uncovered before the auction starts, allowing carriers to take bundling effects of requests into account. More advanced algorithms might further include functions for anticipating the auction progress which is not yet approached.

4.2. Single request reassignment

If requests of the candidate set are reassigned one by one, a second price sealed bid auction, also known as Vickrey Auction, can be employed in Step 4 of the procedure. Using the Vickrey Auction encourages the network members to bid their true value for a request (Vickrey, 1961). Here, the bidder with the highest bid wins the auction and has to pay a price that corresponds to the second highest bid. Hence, sharing the gained profit among the two concerned carriers, as outlined in Step 5, is already included in Step 4.

The example given in Table 1 is continued in order to illustrate the reassignment of requests as coordinated by a Vickrey Auction. For this purpose, we consider two more carriers in the network. We further suppose that both have contracted three transportation requests for the period under consideration. The coordinates of the depots and the transportation requests are again drawn from a TSP instance. The profit calculation of the carriers is shown in Table 2.

The Single Request Reassignment Algorithm (SRRA) tries to reassign request by request such that the period profit of the CCN permanently increases. The inner loop of SRRA is depicted in Fig. 2. Starting from the initial state of the CCN, the carriers first evaluate their requests and then select the least profitable one as a candidate for being sub-contracted to another carrier. Here, Requests 1, 6, and 8 are selected. The individual marginal profits and the pickup and delivery locations are posted by the carriers to a central authority, which composes a single bundle from the candidate set $S = \{1, 6, 8\}$. It consists of the request with the lowest marginal profit in s ($S_1 = \{1\} \subset S$), because it is most likely that this request can be served by another carrier at lower cost.

The current owner of the selected request acts as the auctioneer in Step 3. His marginal profit serves as a floor price in the auction. To reduce the information transparency, the announced floor price is not disclosed by the central authority. According to the marginal profit of Carrier 1, the floor price is set to -8. A floor price of zero indicates that the auctioneer is willing to hand over a request without compensation payment. If the floor price is negative, he also accepts to pay an additional rate to a sub-contractor. In Step 3, the other network members decide on bids for the offered request by including it in their operations planning and computing the marginal profit. Even a negative marginal profit can be posted as a bid to the auctioneer.

The bidder with the highest bid wins the auction in Step 4. In the example it is Carrier 3 with a bid of 25. Sharing the gained profit between Carriers 1 and 3 is realized by a compensation price that Carrier 1 receives from Carrier 3. This price is determined by the second highest bid, i.e. the compensation price for Request 1 is $v_1 = 14$. If all carriers bid below the floor price, the request is not reassigned and remains in the request set of the auctioneer. If only one carrier overbids the floor price, the request is assigned to this carrier and the floor price is taken as the compensation price.

Finally, the authority compares the current period profit of the network with the new period profit. In the example, a profit gain is achieved for Carrier 1 ($\Delta P_1 = 14 - (-8)$) and for Carrier 3 ($\Delta P_3 = 25 - 14$) which leads to a total gain of $\Delta P = 33$. Since $\Delta P > 0$ holds, the reassignment has been realized, i.e. Request 1 leaves N_1 and enters N_3. Then, the procedure is continued in Step 1, where a new candidate set is built. Otherwise, if no improvement of the network profit has been achieved,

Table 2
Profit calculation for carriers $i = 2,\ 3$.

Requests j	Carrier	Revenue		Cost		Profit p_{ij}
		d_j	r_j	l_{ij}	c_{ij}	
4	2	38	96	11	21	75
5	2	39	98	5	15	83
6	2	38	96	25	35	61
7	3	50	120	15	25	95
8	3	18	56	12	22	34
9	3	38	96	14	24	72
		$\sum d_j$	R_i	$L(N_i)$	C_i	P_i
N_2		115	290	154	184	106
N_3		106	272	126	156	116

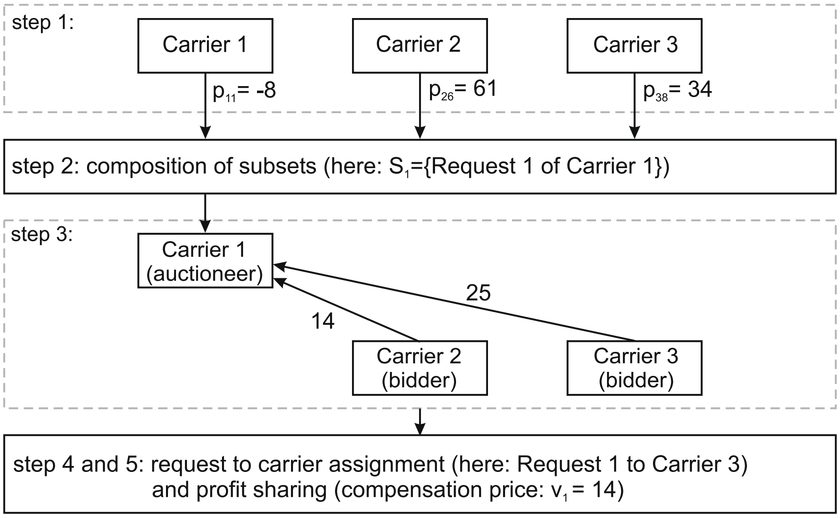

Fig. 2. Sketch of a single request reassignment.

the authority selects a so far unconsidered request from the candidate set for the next auction. SRRA terminates if all requests of the current candidate set have taken part in an auction without obtaining a further increase of the network profit.

4.3. Bundle request reassignment

The economic efficiency of exchange markets can be significantly enhanced through bundled bids (de Vries and Vohra, 2003). Bidding together on multiple requests enables carriers to express specific preferences, e.g. to fulfill some requests in combination or to fulfill a certain request provided that one of their own requests is sub-contracted to another carrier. To allow bidding on request bundles, we incorporate a combinatorial auction in the reassignment procedure. For this end, we consider all subsets of the candidate set as bundles in Step 2. Provided the candidate set contains m requests, at most $2^m - 1$ bundles can be composed from S, where the empty set is neglected.

To determine the winners of a combinatorial auction, the so called Combinatorial Auction Problem (CAP) has to be solved (de Vries and Vohra, 2003). It is defined as follows: Let M denote a set of bidders (carriers), S a set of m items (e.g. one request of each of the m carriers), and $b_i(S_k)$ the bid that bidder $i \in M$ is willing to pay for bundle $S_k \subseteq S$. Furthermore, let $y(S_k, i)$ denote a binary decision variable indicating whether S_k is allocated to $i \in M$ or not. The goal of the CAP is to maximize the cash flow of the auction. As bids represent the marginal profit that carriers can obtain from bundles, this objective maximizes the total profit of the carrier network after the reallocation of requests.

$$\max \quad Z = \sum_{i \in M} \sum_{S_k \subseteq S} b_i(S_k) \cdot y(S_k, i) \tag{12}$$

$$\text{s.t.} \quad 1 \geqslant \sum_{i \in M} \sum_{S_k \ni j} y(S_k, i) \quad (\forall j \in S) \tag{13}$$

$$1 \geqslant \sum_{S_k \subseteq S} y(S_k, i) \quad (\forall i \in M) \tag{14}$$

$$y(S_k, i) \in \{0, 1\} \quad (\forall S_k \subseteq S, \;\; i \in M) \tag{15}$$

Constraints (13) ensure that each item is awarded to a bidder at most once and constraints (14) ensure that no bidder can win more than one of his bids.

Our second algorithm, referred to as Bundle Request Reassignment Algorithm (BRRA), incorporates a combinatorial auction, where the carriers are allowed to bid on every bundle of requests. BRRA is illustrated at the previous example. In Step 1, the algorithm works like SRRA, whereas in Step 2, all subsets of the candidate set $S = \{1, 6, 8\}$ are composed instead of only a single one. This leads to seven bundles $S_1 = \{1\}$, $S_2 = \{6\}$, $S_3 = \{8\}$, $S_4 = \{1, 6\}$, $S_5 = \{1, 8\}$, $S_6 = \{6, 8\}$, and $S_7 = \{1, 6, 8\}$. In Step 3, the carriers determine their marginal profit for each of the bundles. The profit values are communicated to the central authority but not to the other carriers. The central authority considers the received values as bids which are inserted into a bid matrix. According to this algorithmic design, the carriers are bidding independently of the bids of the other carriers. In the considered example the following bid matrix is generated.

$$[b_i(S_k)] = \begin{bmatrix} b_1(S_1) & \dots & b_1(S_k) \\ \vdots & \ddots & \vdots \\ b_m(S_1) & \dots & b_m(S_k) \end{bmatrix} = \begin{bmatrix} -8 & 45 & 8 & 38 & 14 & 61 & 72 \\ 18 & 61 & \mathbf{44} & 75 & 58 & 95 & 106 \\ 25 & 64 & 34 & \mathbf{89} & 59 & 100 & 125 \end{bmatrix}$$

The winners of the auction are determined in Step 4. For this purpose the corresponding CAP is solved. In the example, Carriers 2 and 3 are the winners of the disjoint bundles S_3 and S_4 with the bid $b_2(S_3) = 44$ and the bid $b_3(S_4) = 89$. The corresponding total cash flow is $Z = 44 + 89 = 133$. As the result, Carrier 2 takes over Request 8 from Carrier 3 and sub-contracts his own Request 6 to Carrier 3. Carrier 3 takes over Requests 1 and 6 from Carriers 1 and 2. BRRA terminates if every carrier wins his own request.

In Step 5, the profit gain of the CCN is computed at first. For this purpose the total cash flow is reduced by the bids that the carriers have announced for their own requests (representing the floor prices). In the example we obtain $\Delta P = 133 - (-8) - 61 - 34 = 46$ as the total gain. To split it up among the carriers, we use a simple method. First, the gain is calculated for every bundle by subtracting the floor prices of the contained requests from the winning bid. In the example the gain is $44 - 34 = 10$ for S_3 and $89 - (-8) - 61 = 36$ for S_4. Then, the remaining contribution of the auctioned bundles is divided uniformly among the collaborating carriers. To ensure integer shares, down-rounded values are transferred to the selling carriers and the winner of a bundle obtains the rest. Here, the gain 10 of S_3 is divided among Carriers 2 and 3 in equal shares. Correspondingly, the gain 36 of S_4 is divided among all three carriers. This leads to individual profit gains $\Delta P_1 = 12,\ \Delta P_2 = 17$, and $\Delta P_3 = 17$. Of course, more advanced methods for profit sharing can be employed in Step 5 as well.

It can be seen that the number of bundles considered in BRRA grows exponentially with the size of the candidate set. In order to bound the arising computational effort, several actions can be implemented. In our approach every carrier is allowed to insert only one request in the candidate set at a time. Hence, BRRA can be executed for small collaborative network. For larger networks, or if carriers are allowed to insert more than one request in the candidate set, heuristics must be used to solve the underlying routing problems and the CAP, which is an NP-hard problem as well. A further remedy to the problem is to allow the carriers to place bids only on the most attractive bundles. In such constellations every carrier must place at least one bid on each of his own requests in order to guarantee that the resulting CAP is solvable. Obviously, an incomplete bundle reassignment algorithm can reduce the computational effort drastically at the expense of smaller collaboration gains. This might still yield an efficient performance.

Unfortunately, strategic behavior of carriers is not automatically hindered by a simple combinatorial auction since the assumption about Vickrey Auctions does not hold. Contrasting the Vickrey Auction, in Combinatorial Auctions the bidders determine the compensation price payable to the auctioneer on their own. For this reason a bidder can try to improve its individual profit by announcing bids on requests below its marginal profit. In case that the carrier succeeds, the shares of the collaboration gain of the network partners decrease. However, if the bid is reduced too much, the carrier does not win the auction anymore. The requests will be assigned to other carriers leading to a decrease in the total collaboration gain. This shows, that truthfulness of the bidders is not guaranteed by a combinatorial auction as long as the expected additional profit exceeds the risk of loosing the auction.

To minimize incentives for strategic behavior Varian and Mackie-Mason (1994) have proposed the Generalized Vickrey Auction which extends the combinatorial auction by the Vickrey principle. The procedure ensures an efficient allocation of request bundles where carriers announce their true values as bids. Unfortunately, this method has a high computational effort. If strategic behavior cannot be hindered by an applicable incentive scheme, the above proposed simulation framework might be used to find out when and to what extent untruthfulness pays off for the carriers. However, this issue is not within the scope of this paper. Since we aim at identifying the theoretical potentials for carrier collaboration we can suppose the truthfulness of the carriers in our computations.

5. Computational study

In this section we aim at an assessment of collaboration against the disposition to share customer information and the competition in the market. For this we consider carriers operating from individual depots located within their customer areas. Three geographical classifications are distinguished in which the customer areas of the carriers are adjacent, overlapping, and identical. In case of adjacent customer areas, there is only little competition. The more the areas overlap, the more competition increases. Finally, in case of an identical customer area, there is strong competition but collaboration can create larger benefits.

We study three strategies in which the carriers (a) do not share any of their market information, (b) share it to a certain, self determined extent, and (c), make it entirely transparent to all other carriers. In case (a), each carrier plans and fulfills the requests received for a period alone. With no collaboration involved, the corresponding total profit made by the carriers is denoted as P_{nc}. In case (c), every carrier communicates its complete request set to a confidential authority, where a central planning takes place. The achieved network profit is denoted P_{cp}. For the decentralized planning strategy (b), it is assumed that every carrier selects and transmits requests on an individual basis to the central authority. These requests are reassigned either singly, or in bundles. In the former case, only one of the requests is disclosed per round, while all transmitted requests are disclosed together in the latter case. Supposed that the success of the CCN depends on the degree of information sharing, it is conjectured that the profit achieved through bundled reassignments (P_{br}) exceeds the profit of a possible series of single request reassignments (P_{sr}). More general, $P_{cp} \geqslant P_{br} \geqslant P_{sr} \geqslant P_{nc}$ should hold.

For each strategy we measure two performance indicators: The collaboration gain $\phi = \frac{P - P_{nc}}{P_{nc}}$ measures the relative gap of the network profit P for a strategy against the profit achievable without collaboration. The decentralization cost expresses the loss due to decentralized planning. It is determines by $\varphi = \frac{P_{cp}-P}{P_{cp}}$, i.e. by the relative distance of the network profit for a strategy against central planning.

To compare the different strategies, we consider a small CCN consisting of three carriers. Initially, every carrier holds a set of three transportation requests. This setting is chosen to assure that we can quickly solve all optimization problems arising in the competing strategies to optimality. The decentralized approaches SRRA and BRRA are implemented in Java 1.6. Instances of the TSPPD are solved by the branch and cut algorithm of Dumitrescu et al. (2007), while CAP instances created by BRRA are solved with the MIP-solver CPLEX 10.0. As outlined above, the central planning strategy leads an MDTSPPD, for which instances with up to ten requests can be solved by our MIP-Solver in two hour computation time.

To derive an unbiased platform for the test, we generate a sufficiently large set of instances from the Euclidean TSP R101 of Solomon (2005). The contained 101 cities are plotted in Fig. 3. Locations 10, 54, and 93 are chosen as the carrier's depots. The total area is divided into three disjoint subsets $X_1,\ X_2$, and X_3 of comparable customer demand. In correspondence to adjacent, overlapping, and identical customer areas, we generate three instance Sets A, O, and I, with 30 test instances each. For instance Set A, the transportation requests are composed from pickup and delivery locations which are randomly drawn for Carrier i from set X_i. For instance Set O, all cities inside the dashed triangle of Fig. 3 are additionally included in each customer area. Finally, for instance Set I, the pickup and delivery locations are drawn arbitrarily for every carrier from the compound set $\cup X_i$.

Each of the 90 test instances is solved in four ways, namely without collaboration, by SRRA and by BRRA, and finally to optimality, using the central planning approach. The corresponding network profits are shown in Table 3. For adjacent customer areas, the profit achieved with no collaboration is hardly improved by the decentralized planning approaches. SRRA and BRRA deliver always the same results. In some instances, the central planning appears advantageous. Regarding overlapping and identical customer areas, the decentralized methods are superior to a solution gathered with no collaboration but are often dominated by the central solution. In most cases, BRRA is better than or as least as good as SRRA. However, $P_{br} \geqslant P_{sr}$ does not strictly hold as verified, e.g. by instance #21 of Set O. This is explained as follows. Both SRRA and BRRA iteratively perform a series of auctions until no further improvement of the network profit is possible. This procedure can be considered as a local search moving from one solution to a neighboring solution until a local optimum is reached. The neighborhood explored by BRRA is larger and therefore finding better solutions than SRRA is likely. However, by chance SRRA can follow a more favorable search trajectory than BRRA leading into an area of the search space where better solutions are located. From this perspective both procedures are merely heuristics where BRRA is more powerful on average.

Table 4 shows these results in aggregated form. It is interesting to aware that there is quite a lot potential for collaboration, even for instance Set A. The collaboration gain ϕ of the MDTSPPD solution is about 19%. However, SRRA and BRRA

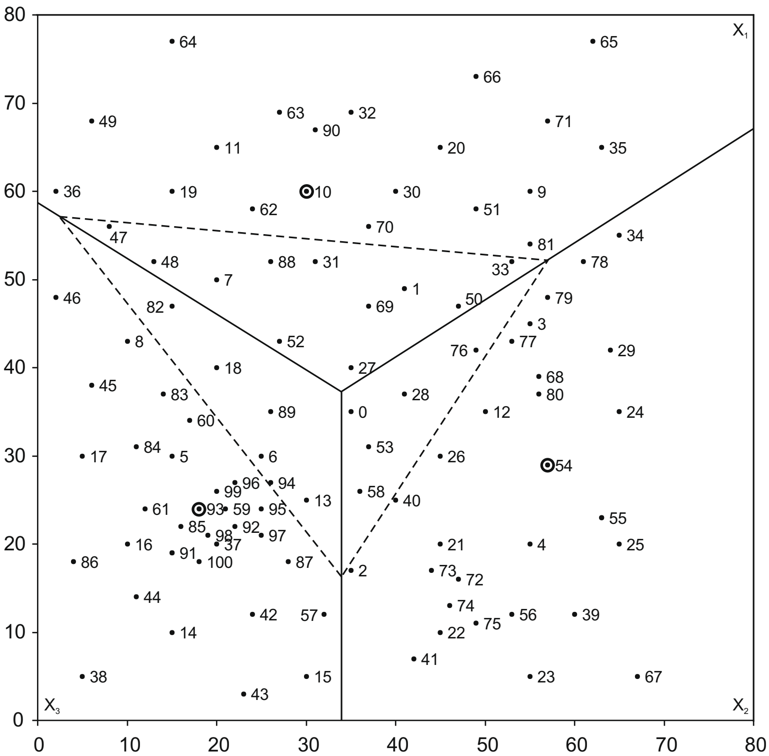

Fig. 3. Generating test instances from Solomons R101 Problem.

Table 3
Profit obtained by the strategies in the three instance sets.

#	Set *A*				Set *O*				Set *I*			
	P_{nc}	P_{sr}	P_{br}	P_{cp}	P_{nc}	P_{sr}	P_{br}	P_{cp}	P_{nc}	P_{sr}	P_{br}	P_{cp}
1	21	21	21	25	273	273	273	328	139	268	320	320
2	164	164	164	216	121	187	187	236	102	272	187	373
3	210	210	210	253	107	164	164	231	142	265	265	367
4	187	187	187	187	138	247	294	303	91	158	158	227
5	112	149	149	149	167	226	231	282	176	189	189	403
6	141	190	190	209	198	198	198	280	141	307	378	439
7	237	237	237	300	176	176	176	240	283	283	283	491
8	218	218	218	229	213	213	213	340	234	288	417	483
9	119	142	142	159	66	137	169	183	170	259	258	355
10	230	230	230	289	139	184	184	286	202	314	237	398
11	182	182	182	201	209	211	211	269	158	332	382	478
12	141	141	141	184	253	282	282	385	84	128	168	346
13	156	156	156	172	201	222	222	236	171	228	228	430
14	105	105	105	105	215	257	257	360	303	303	327	553
15	203	203	203	203	160	212	212	244	163	366	464	464
16	208	208	208	243	186	241	241	325	343	458	573	573
17	156	156	156	176	161	228	228	294	210	338	338	427
18	109	109	109	117	162	173	199	230	224	289	363	440
19	171	182	182	202	123	155	172	211	164	307	408	413
20	164	164	164	191	241	241	241	311	239	414	418	533
21	118	119	119	141	160	255	212	319	61	61	302	302
22	167	178	178	209	219	280	280	300	52	185	293	314
23	140	140	140	143	87	114	114	143	253	559	582	582
24	119	121	121	131	165	237	237	307	203	358	403	453
25	182	182	182	220	158	210	205	286	237	492	492	520
26	176	176	176	255	155	229	229	286	281	465	465	500
27	155	157	157	201	303	349	349	452	185	406	406	487
28	203	203	203	225	232	303	255	385	312	331	482	585
29	162	162	162	193	344	344	344	465	340	459	459	541
30	169	169	169	209	288	378	378	463	256	527	527	527

Table 4
Average-in-set performance and computational effort.

Set	MDTSPPD		SRRA				BRRA			
	ϕ	sec	ϕ	φ	Iteration	sec	ϕ	φ	Iteration	sec
A	0.1907	487.39	0.0347	0.1257	1.33	0.41	0.0470	0.1257	1.33	0.60
O	0.6738	5003.01	0.2869	0.2273	2.83	3.72	0.3084	0.2199	2.40	2.59
I	1.5566	6467.57	0.7664	0.2737	4.87	11.61	1.0088	0.2190	3.97	4.81

merely achieve 3–4% of the possible gain. Consequently, the decentralization cost φ is quite high. The potential for collaboration strongly increases for instance Set O. Here, the collaboration gain of the MDTSPPD solution is already 67%, of which SRRA can accomplish 26% and BRRA even more than 30%. Considering instance Set I, the gap between the theoretic limit and the achieved collaboration gain of the decentralized planning approaches reduces again. Central planning achieves a gain of 155%, SRRA of 76% and BRRA of more than 100%. It can also be seen that the cost of decentralization gets more or less stationary at 22% for BRRA, while the collaboration potential grows constantly from instance Set O to Set I. Obviously, a decentralized planning approach is able to absorb already the bigger part of the possible gain of a collaboration among carriers.

Taking a look at the computational effort of the different strategies, the results can be retraced as well. Table 4 shows the number of auction rounds (iterations) taken by the decentralized planning methods. While SRRA and BRRA perform hardly more than a single round for instance Set A, up to four rounds are taken for Sets O and I. Keep in mind that each round performed guarantees an improvement of the network profit. This is also reflected by the run times, reported for a PC P4 2800 MHz and measured in seconds. BRRA turns out a little faster than SRRA because the effort spend on solving the CAP is negligible for a small candidate.

6. Conclusions

The collaborative carrier routing problem arises in competitive environments where carriers are willing to disclose private information only in case of win-win situations. To overcome the shortcoming of an individual planning of the carriers, a decentralized approach based a confidential exchange of relevant information is proposed. It enables carriers to fix the terms for disclosing information and subcontracting transportation requests independently. On this basis, procedures for iteratively reassigning customer requests can be implemented by using well known auction mechanisms, like the Vickrey Auction or a combinatorial auction. To keep the framework simple, we have made several limitations, like ignoring load capacity and considering only one request per carrier per round. This allows us to apply exact methods for solving the involved optimization problems to optimality and thus compute the true cost of decentralization.

The main contribution of the paper to the literature is to provide a framework for accessing the utility of collaboration on a quantitative basis. The framework enables us to determine the true cost of decentralization against a central planning approach as well as a setting without collaboration. The done computations give evidence to the following findings: Supposed the carriers provide true information, there is significant potential for collaboration approaches to improve the network profit against an individual planning and execution. Decentralized planning clearly diminishes the drawback of individual planning, although the cost of decentralization remains considerable. The only way to reduce these costs is to widen the amount of centrally known data. This is indicated by the fact that bundled reassignments outperform single reassignments on average. The conducted experiments also verify that the more carriers compete within a customer area, the more benefit collaboration produces. All this motivates further research in electronic exchange systems for transportation requests.

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References

Caplice, C., Sheffi, Y., 2003. Optimization based procurement for transportation services. Journal of Business Logistics 24 (2), 109–128.
de Vries, S., Vohra, R.V., 2003. Combinatorial auctions: a survey. INFORMS Journal on Computing 15 (3), 284–309.
Dumitrescu, I., Ropke, S., Cordeau, J.-F., Laporte, G., 2007. Traveling salesman problem with pickup and deliveries: polyhedral results and a branch-and-cut algorithm. Technical report.
Elmaghraby, W., Keskinocak, P., 2003. Combinatorial auctions in procurement. In: Billington, C., Harrison, T.P., Lee, H.L., Neale, J.J. (Eds.), The Practice of Supply Chain Management, International Series in Operations Research & Management Science, vol. 62. Kluwer (Chapter 15).
Ergun, Ö., Kuyzu, G., Savelsbergh, M.Shipper collaboration, 2007. Computers & Operations Research 34 (9), 1551–1560.
Fischer, K., Müller, J.P., Pischel, M., 1996. Cooperative transportation scheduling: an application domain for DAI. Applied Artificial Intelligence 10 (1), 1–33.
Gendreau, M., Potvin, J.-Y., 1998. Dynamic vehicle routing and dispatching. In: Crainic, T.G., Laporte, G. (Eds.), Fleet Management and Logistics. Kluwer Academic Publishers, Boston, Dortrecht, London, pp. 115–126 (Chapter 5).
Gomber, P., Schmidt, C., Weinhardt, Chr., 1999. Efficiency, incentives, and computational tractability in mas-coordination. International Journal of Cooperative Information Systems 8 (1), 1–14.
Klaus, P., 2003. Die "TOP 100" der Logistik: Berichtszeitraum 2001/2002.
Krajewska, M.A., Kopfer, H., 2006. Collaborating freight forwarding enterprises. OR Spectrum 28 (2), 301–317.
Ledyard, J., Olson, M., Porter, D., Swanson, J., Torma, D., 2002. The first use of a combined-value auction for transportation services. Interfaces 32 (5), 4–12.
Lee, C.G., Kwon, R.H., Ma, Z., 2007. A carrier's optimal bid generation problem in combinatorial auctions for transportation procurement. Transportation Research Part E 43 (2), 173–191.
Lu, Q., Dessouky, M., 2004. An exact algorithm for the multiple vehicle pickup and delivery problem. Transportation Science 38 (4), 503–514.
McAfee, R.P., McMillan, J., 1987. Auctions and biddings. Journal of Economic Literature 25 (2), 699–738.
Mitrovic-Minic, S., Krishnamurti, R., Laporte, G., 2004. Double-horizon based heuristics for the dynamic pickup and delivery problem with time windows. Transportation Research Part B 38, 669–685.
Sandholm, T., 1993. An implementation of the contract net protocol based on marginal cost calculation. In: Proceedings of the 11th National Conference on Artificial Intelligence (AAAI-93). Whashington, DC, pp. 256–263.
Schönberger, J., 2005. Operational Freight Carrier Planning. Springer.
Shapley, L.S., 1953. A value for n-person games. In: Kuhn, H.W., Tucker, A.W. (Eds.), Contributions to the Theory of Games, Annals of Mathematics Studies, vol. II. Princeton University Press, Princeton, pp. 307–317.
Sheffi, Y., 2004. Combinatorial auctions in the procurement of transportation services. Interfaces 34 (4), 245–252.
Shen, Y., Potvin, J.-Y., Rousseau, J.-M., Roy, S., 1995. A computer assistant for vehicle dispatching with learning capabilities. Annals of Operations Research 61, 189–211.
Solomon, M.M., 2005. <http://w.cba.neu.edu/%7Emsolomon/problems.htm> (accessed 29.06.07).
Song, J., Regan, A.C., 2002. Combinatorial auctions for transportation service procurement: the carrier perspective. Transportation Research Record, Journal of the Transportation Research Board, 40–46. 1833.
Song, J., Regan, A.C., 2005. Approximation algorithms for the bid construction problem in combinatorial auctions for the procurement of freight transportation contracts. Transportation Research Part B 39 (10), 914–933.
Turban, E., Leidner, D., McLean, E., Wetherbe, J. (Eds.), 2005. Information Technology for Management. Wiley.
Varian, H.R., Mackie-Mason, J.K., 1994. Generalized vickrey auctions. Available at SSRN: <http://ssrn.com/abstract=975873>.
Vickrey, W., 1961. Counterspeculation, auctions, and competitive sealed tenders. The Journal of Finance 16 (1), 8–37.