

# Multicriteria Tour Planning for Mobile Healthcare Facilities in a Developing Country

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## Abstract

A multiobjective combinatorial optimization (MOCO) formulation for the following location-routing problem in healthcare management is given: For a mobile healthcare facility, a closed tour with stops selected from a given set of population nodes has to be found. Tours are evaluated according to three criteria: (i) an economic efficiency criterion related to the tour length, (ii) the criterion of average distances to the nearest tour stops corresponding to  $p$ -median location problem formulations, and (iii) a coverage criterion measuring the percentage of the population unable to reach a tour stop within a predefined maximum distance. Three algorithms to compute approximations to the set of Pareto-efficient solutions of the described MOCO problem are developed. The first uses the P-ACO technique, and the second and the third use the VEGA and the MOGA variant of multiobjective genetic algorithms, respectively. Computational experiments for the Thiès region in Senegal were carried out to evaluate the three approaches on real-world problem instances.

**Keywords:** Facility location, metaheuristics, mobile healthcare, multicriteria decision making, routing.

## 1 Introduction

Developing countries frequently face the dilemma of very restrictive budget limitations for healthcare expenditures and a growing population. In such a situation,

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the provision of cost-effective healthcare facilities becomes particularly important. Distance proved to be one of the most influencing factors for the utilization of healthcare facilities (see [36], [45], [7], [47]). In developed countries and mainly in urban areas, distance rather influences the decision on which kind of medical services (e.g., a medical doctor or a hospital) the patients use [34], whereas in rural areas of developing countries, distance is the decisive factor whether or not to use medical services at all [38]. Therefore, in these regions, the provision of medical facilities close to the residences of the people becomes crucial for appropriate medical supply.

As a possible way to provide cost-effective primary healthcare under the very restrictive budget limitations of a developing country (cf. Flessa [18]), some governments and institutions have supplemented hospitals and stationary dispensaries with *mobile healthcare facilities* (see Gilson [23], Foord [20], Fox-Rushby [21], Mackle and Giles [33], Ruggiero and Gloyd [43], Dyer [17], or Hodgson et al. [28]). One of the most salient purposes of such mobile facilities lies in the extension of *access* of people to health services. Achieving the same accessibility effect by building a larger number of spatially fixed healthcare units would increase the costs for equipment and staff considerably, which often cannot be afforded. Small mobile units, on the other hand, are able to travel to distinct places at distinct times and to offer service for the people in a certain radius.

Obviously, mobile units cannot make *all* the services available a hospital can offer. Thus, they can be seen as supplemental to other medical services, satisfying either the most urgent needs, or providing services of certain specialized medical divisions like dental treatment [3], the expertise of eye specialists [43], or CT scanners [46]. Of course, in the situation of a developing country, which is considered in this article, the medical equipment will usually not meet very advanced standards [48], but basic medical services can be offered at a high quality level.

Already for fixed facilities, the question where they should be built and how they should be staffed is a difficult planning problem. Several types of location-allocation models aim at a decision support for this question based on quantitative data (see, e.g., Berghmans et al. [1], Hindle and Ngwube [27], Oppong [39], Doherty et al. [14], Mehrez et al. [35], Chu and Chu [5], Rahman and Smith [41], Goldstein et al. [24] or Galvao et al. [22]). In the case of one or more *mobile* healthcare facilities, the planning problem gets even more complex since both tours and stops on tours have to be selected in a way that satisfies different criteria, cost-effectiveness (influenced by travel distances) being one of them, accessibility and coverage being others.

Hodgson et al. [28] and Hachicha et al. [26] have addressed the tour planning problem for one or more mobile facilities in the Suhum district in Ghana. In their optimization model, tours and stops on tours are computed from geographical and demographic data both for the road conditions in the dry and in the rainy season. As

a coverage constraint, the authors demand that each population center (settlement) that can, in principle, be reached within a given maximum walking distance (the cases of three and of eight kilometers are considered) is actually provided by a tour stop within this distance.

In the literature, the considered distance limits for such coverage constraints vary significantly depending on the concrete situation. Berghmans et al. [1], for instance, suggested a maximum walking distance of 750 meters to the nearest health center, whereas Patel [40] suggested in his model for the rural region Dharampour in India a maximum walking time of 1.5 hours. In any case, compared to standards in Europe or the USA, where a maximum walking or driving time of 15 minutes for more than 90 percent of the population is strived for, the limits for rural areas of developing countries must necessarily be set to a far less ambitious level.

In the present article, we extend the model in [28] and [26] to a *multiobjective* problem formulation: Whereas in the indicated articles, an optimization problem with *tour length* as the objective function is solved, we do not judge the quality of a tour plan only based on a single criterion, but rather take account of the multi-criteria nature of the task (see below) and intend to provide the political decision maker with a computer-based decision support system (DSS) which outputs several candidate solutions for final choice. They are visualized and can be evaluated and discussed on a political level. So, the final decision remains up to the human decision maker, but the system assists him/her in coping with the complexity of the problem. Literature examples show that decision makers in the governmental departments seldom transformed the results of healthcare location/allocation models *unchanged* into concrete policies (cf., e.g., [2], [10], [25], [35]). Already for this reason, it seems advisable to integrate the decision makers as early as possible into the solution procedure.

In the case of our problem, at least the following three criteria should be taken into consideration (possibly even more): (1) *Effectiveness of workforce employment*, measured by the ratio between medical working time and total working time including travel time and facility setup time. (2) *Average accessibility*, measured by a low average time required by the inhabitants of the considered region to reach the nearest tour stop or the nearest stationary facility. (3) *Coverage*, expressed by the percentage of inhabitants living within a given maximum walking distance to a tour stop or stationary facility. In some sense, this definition of coverage aims at the aspect of equity (or fairness) of accessibility: As far as possible, no citizen should be excluded at all from medical services by an extraordinary distance to the nearest facility.

Let us mention that our criterion (1), effectiveness, will turn out as closely related to the *tour length* criterion which is the objective in the well-known classical *routing*

problems TSP and VRP. The two other criteria (2) and (3), on the other hand, refer to the *location* aspect of the problem: Optimizing only the average accessibility (2) would amount to solving a *p-Median Problem* (see, e.g., ReVelle and Swain [42]), while optimizing only the coverage (3) would mean that a *Maximal Covering Location Problem* (MCLP), as formulated by Church and ReVelle [6], is solved.

There are tradeoffs between the three criteria above: Evidently, effectiveness can be increased by reducing the number of stops, leading to a reduced average accessibility or coverage. Vice versa, average accessibility and coverage can be increased by increasing the number of stops, which reduces effectiveness. Also average accessibility and coverage contradict to some extent: A low overall average walking distance can be achieved by leading the tour mainly through areas with dense population and planning a large number of stops there, which, however, effects an inequitable solution with comparably low coverage. Vice versa, to achieve high coverage, tour stops must be spread broadly over the whole region, which increases distances in those parts that “count most” from the viewpoint of average distances, namely the densely populated areas.

A model for the problem under consideration will be presented in Section 2. Section 3 presents solution algorithms. In Section 4, we shall study the application of these algorithms to the tour planning problem for the Thiès region in Senegal. Section 5 contains concluding remarks.

## 2 The Model

Let us restrict ourselves to the case of one single mobile facility (MF). Moreover, we assume here that medical supply for the considered region is to be delivered exclusively by the MF, without support by fixed hospitals or dispensaries. This is, of course, a simplification that usually does not represent the real situation, not even in a country with low healthcare standards. Also in the region of Senegal to which our computational example in section 4 refers, fixed healthcare facilities exist. Nevertheless, for the sake of a better isolation of the methodological questions raised by the considered location-routing problem, it is convenient to start with the mentioned assumption. The extension of the model to the more realistic situation of combined stationary and mobile supply is discussed in section 5; in our opinion, this extension is rather straightforward.

We use the following formal model description to represent the problem:

As in [28], a problem instance is based on a graph  $G = (W, E)$ , where the nodes  $v_i \in W$  are settlements (population centers of any kind, from cities to very small villages), and the edges  $e_l \in E$  are traffic links (roads or paths) between these

settlements. An edge  $e_i$  can be represented as the pair  $(v_i, v_j)$  of the two incident nodes. In each settlement  $v_i \in W$ , there lives a population of  $p_i$  inhabitants. The sum of the values  $p_i$  is the total number of inhabitants,  $N$ .

A subset  $V \subseteq W$  contains the potential stops of the MF. Without loss of generality, the nodes  $v_i \in W$  can be labelled in such a way that the nodes in  $V$  get the lowest indices:  $V = \{v_1, \dots, v_{|V|}\}$  and  $W = \{v_1, \dots, v_{|W|}\}$  with  $|V|$  and  $|W| \geq |V|$  denoting the number of elements in  $V$  and  $W$ , respectively. The shortest distance between two nodes  $v_i \in W$  and  $v_j \in W$  is  $d_{ij}$  kilometers, the shortest driving time of the MF between two nodes  $v_i \in V$  and  $v_j \in V$  is  $c_{ij}$  hours. (Hodgson et al. [28] comprise these two types of variables to a single variable. With respect to different quality types of roads, however, it makes sense to consider them separately from each other.)

The time interval during which the MF performs its (closed) tour is called a *period*. The number of days of a period is considered as a given constant fixed in advance. It forms an aspect of the quality of service and should not be fixed at a too high value, otherwise continuity of medical treatment would not be guaranteed.

The decision variable is the chosen (closed) tour,

$$\pi = (\pi(1), \dots, \pi(k)),$$

where  $\pi(j)$  is the index of the  $j$ th visited node ( $v_{\pi(j)} \in V$ ;  $j = 1, \dots, k$ ), and after visiting node  $\pi(k)$ , the MF returns to the start node  $\pi(1)$ . This start node is a fixed given depot; it is always possible to choose the indices of the nodes in such a way that  $\pi(1) = 1$ . The reader should be aware that, contrary to the well-known travelling salesperson problem (TSP) or to most types of vehicle routing problems (VRP), not every node  $v_i \in V$  needs to be part of the tour.

The number of stops on the tour is  $k = k(\pi)$ . Thus, the total driving time during the tour is given by

$$t(\pi) = \sum_{j=1}^{k-1} c_{\pi(j), \pi(j+1)} + c_{\pi(k), 1}.$$

The following constant parameters are used as input data:

- $T$ : total working time of a member of the MF personnel during the period (expressed in hours),
- $\mu$ : time for the setup of the MF at a stop per member of the MF personnel (expressed in hours),
- $M$ : upper bound for an acceptable walking distance to the nearest tour stop (in kilometers), as defined by the political decision maker.

The three objective criteria are formulated in terms of *costs* in the way described below.

*Objective (1): Effectiveness of workforce employment*

The ineffectiveness of the MF personnel employment is measured by the ratio of medically non-productive time (the required time for the setup of the MF at a certain location, plus the driving time between locations) to the overall working time,

$$\frac{\mu k(\pi) + t(\pi)}{T}. \quad (1)$$

Therefore, our first objective is a weighted average of number of stops and tour length: With  $\gamma_1 = \mu/T$  and  $\gamma_2 = 1/T$ ,

$$Z_1 = \gamma_1 k(\pi) + \gamma_2 t(\pi). \quad (2)$$

It is also possible to interpret  $Z_1$  exclusively as a tour length: By setting  $c'_{ij} = (c_{ij} + \mu)/T$ , one obtains  $Z_1$  as the tour length with respect to distances  $c'_{ij}$ .

Both the nominator and the denominator of (1) are expressed in hours, such that  $Z_1$  becomes a dimensionless number (between 0 and 1).

*Objective (2): Average accessibility*

Average accessibility can be measured by computing the average distance a member of the population has to walk in order to reach the nearest stop of the MF. (Evidently, this is a criterion which corresponds to the classical  $p$ -median formulation of location problems.) Thus, we set

$$Z_2 = \frac{1}{N} \sum_{v_i \in W} p_i d(i, \pi), \quad (3)$$

where

$$d(i, \pi) = \min\{d_{i, \pi(j)} \mid j = 1, \dots, k\}$$

is the minimum distance of  $v_i$  to a node contained in the tour  $\pi$ .

Objective function value  $Z_2$  is expressed in kilometers as distance units.

*Objective (3): Coverage*

To measure the aspect of coverage, we introduce as a third objective function the share of the population living in a distance larger than the pre-defined value  $M$  to the nearest tour stop. Expressed in formulas:

$$Z_3 = \frac{1}{N} \sum_{v_i \in W(M, \pi)} p_i, \quad (4)$$

where  $W(M, \pi)$  is the set of all nodes  $v_i \in W$  with  $d(i, \pi) > M$ .

Objective function value  $Z_3$  is a ratio between numbers of inhabitants and hence a dimensionless number (between 0 and 1).

We can give a *three-objective integer linear programming* (ILP) formulation of our problem (2) – (4): Let us re-encode the combinatorial decision variable  $\pi$  by introducing the integer variables

$$x_{ij} = \begin{cases} 1, & \text{if } v_j \text{ is immediate successor of } v_i \text{ on tour } \pi, \\ 0, & \text{otherwise,} \end{cases}$$

for  $v_i, v_j \in V$ . Moreover, we introduce the additional integer variables

$$y_i = \begin{cases} 1, & \text{if } v_i \text{ is selected as a tour stop (i.e., element of } \pi), \\ 0, & \text{otherwise,} \end{cases}$$

for  $v_i \in V$ , the integer variables

$$z_{ij} = \begin{cases} 1, & \text{if population node } v_i \text{ is supplied by a stop in } v_j, \\ 0, & \text{otherwise,} \end{cases}$$

for  $v_i \in W, v_j \in V$ , and the integer variables

$$u_i = \begin{cases} 1, & \text{if population node } v_i \text{ is covered within distance } M, \\ 0, & \text{otherwise,} \end{cases}$$

for  $v_i \in W$ . Finally, we define the coverage matrix  $A = (a_{ij})$  by

$$a_{ij} = \begin{cases} 1, & \text{if } d_{ij} \leq M, \\ 0, & \text{otherwise,} \end{cases}$$

for  $v_i \in W, v_j \in V$ . Then, an equivalent representation of our problem is given by:

$$\min \left( \sum_{v_i, v_j \in V, i \neq j} c'_{ij} x_{ij}, \sum_{v_i \in W} p_i \sum_{v_j \in V} d_{ij} z_{ij}, - \sum_{v_i \in W} w_i u_i \right) \quad (5)$$

s.t.

$$\sum_{v_j \in V} x_{ij} = y_i \quad (v_i \in V) \quad (6)$$

$$\sum_{v_i \in V} x_{ij} = y_j \quad (v_j \in V) \quad (7)$$

$$\sum_{v_i \in S, v_j \in V \setminus S} x_{ij} \geq y_t \quad (S \subset V, v_1 \notin S, v_t \in S) \quad (8)$$

$$\sum_{v_j \in V} z_{ij} = 1 \quad (v_i \in W) \quad (9)$$

$$y_j - z_{ij} \geq 0 \quad (v_i \in W, v_j \in V) \quad (10)$$

$$\sum_{v_j \in V} a_{ij} y_j \geq u_i \quad (v_i \in W) \quad (11)$$

$$x_{ij} \in \{0, 1\} \quad (v_i \in V, v_j \in V) \quad (12)$$

$$z_{ij} \in \{0, 1\} \quad (v_i \in W, v_j \in V) \quad (13)$$

$$u_i \in \{0, 1\} \quad (v_i \in W) \quad (14)$$

The first component of (5) is objective function  $Z_1$ , represented by means of the modified costs  $c'_{ij}$  introduced after eq. (2). The second component is objective function  $Z_2$ , multiplied by  $N$ . The third component is obtained from objective function  $Z_3$  by first multiplying  $Z_3$  by  $N$ , and then subtracting the constant  $N$  from the result.

Conditions (6) and (7) ensure that every node on the tour has exactly one successor and one predecessor, and that nodes outside the tour have neither successors nor predecessors. Conditions (8) are the usual subtour elimination constraints for the TSP, applied to tour stops. Conditions (9) and (10) together ensure that every population node is supplied by a tour stop. Conditions (11) state that a population node can only be covered within distance  $M$  by a node if this node is within distance  $M$  and chosen as a tour stop. Conditions (12) to (14), finally, are the usual binary integrality constraints.

We would like to emphasize that the given formulation of the multiobjective location-routing problem with objectives (5) – (14) is general enough to be relevant also for other areas of application.

### 3 Solution Algorithms

Our aim is to provide the decision maker with a set of feasible solutions of the multicriteria optimization problem by eliminating those solutions that (a) are not Pareto-efficient, or (b) violate some predefined aspiration levels. A solution is called *Pareto-efficient* if there is no other solution that dominates it, where the dominance relation is defined as follows: solution  $\pi_1$  *dominates* solution  $\pi_2$ , if  $\pi_1$  is at least equally good as  $\pi_2$  with respect to all objective functions, and better than  $\pi_2$  with



respect to at least one objective function. An *aspiration level* is an upper bound for some objective (cost) function.

The exclusion of solutions violating aspiration levels becomes trivial as soon as the set of Pareto-efficient solutions has been determined (except in the case where this set is too large to be stored). Therefore, our real concern is the determination of Pareto-efficient solutions.

For problem instances of a realistic magnitude, an algorithm computing *exactly* the set of all Pareto-efficient solutions cannot be expected. This is due to the fact that the problem under consideration is NP-hard: Consider the special case where  $\mu = 0$  and  $M$  is smaller than  $\min_{i \neq j} d_{ij}$ . A solution minimizing  $(Z_3, Z_1, Z_2)$  lexicographically (i.e., giving  $Z_1$  a negligible weight compared to  $Z_3$ , and  $Z_2$  a negligible weight compared to  $Z_1$ ) is Pareto-efficient and must therefore occur in the desired output set as an identifiable element. However,  $Z_3$  is minimized in the indicated special case by choosing *all* elements of  $V$  as tour stops, and given that this is done,  $Z_1$  is minimized by solving the TSP with distance matrix  $c_{ij}$ . The solution of a TSP, however, is an NP-hard problem. Therefore, also (5) – (14) is an NP-hard problem, which makes the application of *heuristics* to its approximate solution advisable.

For this purpose, we have designed and implemented two approaches, which will be presented in the sequel.

### 3.1 Approach 1: Simultaneous Location and Routing by P-ACO

Our first approach treats the location aspect and the routing aspect of the problem simultaneously. For this purpose, we have adapted the P-ACO technique, a multicriteria metaheuristic introduced in [12] and [13], to the problem considered here. P-ACO (Pareto-ACO) generalizes the Ant Colony Optimization (ACO) metaheuristic (see below) for single-objective problems to the case of several objective functions, determining approximations to the set of Pareto-efficient solutions.

The ACO approach has been developed since 1992 by Dorigo, Maniezzo and Coloni and has found numerous applications in diverse fields in the meantime. For surveys, we refer the interested reader to [16] or [15]. Let us shortly recapitulate the basic ideas of the approach. ACO is a nature-inspired metaheuristic with the following main features:

- Solutions are constructed randomly and step-by-step.
- Construction steps that have *turned out* as part of good solutions are favored.

- Construction steps that can be *expected* to be part of good solutions are favored as well.

The stepwise construction of a solution is represented by a random walk in a certain graph. One imagines that these walks are performed by conceptual units called *ants*.

In our application, the graph on which the random walks take place is a complete undirected graph with the settlements  $v_i \in V$  as nodes and an edge between each pair of nodes. For the sake of a more concise notation, we shall refer to node  $v_i$  simply by  $i$  in this subsection.

There are problem-dependent rules determining for each step of the walk which nodes are feasible as successor nodes and which are not. When there is no feasible successor node anymore or some stopping criterion is satisfied, the walk ends and is decoded as a complete solution of the problem.

In our application, the walk always starts at node 1 of the graph, the feasibility rule says that no node except the start node is allowed to be visited more than once, and the stopping criterion is that the start node 1 is visited for the second time.

When constructing a walk, the probability to go from a node  $i$  to a feasible successor node  $j$  is chosen as

$$Pr(j|i) \text{ proportional to } \tau(i, j) \cdot \eta(i, j), \quad (15)$$

where  $\tau(i, j)$  is the so-called *pheromone value*, a memory value storing how good step  $(i, j)$  has been, and  $\eta(i, j) = \eta_u(i, j)$  is the so-called *visibility*, a pre-evaluation of how good step  $(i, j)$  will presumably be, given the partial walk  $u$  up to now. This pre-evaluation is done in a problem-specific manner. Pheromone initialization and update is performed as follows:

*Pheromone initialization:* Set  $\tau(i, j) = 1$  for all edges  $(i, j)$ .

*Pheromone update:* First, set, for each edge  $(i, j)$ ,

$$\tau(i, j) = (1 - \rho) \tau(i, j)$$

where  $\rho$  is a so-called *evaporation factor* between 0 and 1. This step is called *evaporation*. Then, the pheromone values on one or more “good” paths found up to now are reinforced by pheromone increments (possibly of different size). How this is done in detail depends on the specifically chosen pheromone update strategy; different strategies are described in the ACO literature. For the present paper, we used the rank-based update strategy of [4] which we will outline below.

Random walk construction and pheromone update are iterated. Usually, instead of only a single walk (“one ant”),  $s$  walks ( $s > 1$ ) are constructed sequentially

or (in parallel implementations) simultaneously (“ $s$  ants”). The *rank-based* update strategy applied in this paper consists in selecting the  $s_0$  best walks out of the  $s$  walks generated in the current round, where  $s_0 < s$  is a constant, and in adding an increment

$$\Delta_l = \delta_0 \cdot (s_0 - l + 1)$$

to the pheromone value of each edge of the  $l$ th-best walk,  $l = 1, \dots, s_0$ , with a suitable increment factor  $\delta_0 > 0$ . We call the iteration consisting of the  $s$  random walks of the ants, plus the following pheromone update, a *round*. In total, a fixed number  $R$  of rounds is executed.

P-ACO extends ACO (i) by an additional outer iteration in which random weights for each objective function are chosen, (ii) by checks whether a newly found solution is non-dominated by candidate solutions in a current solution set and vice versa, and (iii) by a more refined pheromone handling mechanism.

For our problem, we realize the P-ACO approach as follows: A set  $\Pi$  of tours is initialized as the empty set. In successive iterations called *periods*, weights  $w_1$ ,  $w_2$  and  $w_3$  for the objective functions  $Z_1$ ,  $Z_2$  and  $Z_3$ , respectively, are drawn randomly. Within each period, for the given weight vector  $w = (w_1, w_2, w_3)$ , a heuristic optimization run for the single-objective problem

$$w_1 Z_1 + w_2 Z_2 + w_3 Z_3 \rightarrow \min$$

is performed using ACO. If the resulting tour is non-dominated by all tours in  $\Pi$  with respect to the three objective functions (i.e., if there is no tour in  $\Pi$  that performs at least equally well as  $\pi$  with respect to  $Z_1$ ,  $Z_2$  and  $Z_3$ , and even better than  $\pi$  with respect to at least one objective function), then  $\pi$  is added to  $\Pi$ . If, in this case, certain solutions in  $\Pi$  are dominated by  $\pi$ , they are deleted from  $\Pi$ . After termination of the loop over the periods,  $\Pi$  contains the suggested solution candidates. In total, a fixed number  $Q$  of periods is executed.

In principle, pheromone values are assigned and modified as in the standard ACO algorithm, but we distinguish between specific pheromone values  $\tau_1(i, j)$ ,  $\tau_2(i, j)$  and  $\tau_3(i, j)$  for  $Z_1$ ,  $Z_2$  and  $Z_3$ , respectively. Pheromone increments are performed in proportion to  $w_1$ ,  $w_2$  and  $w_3$ , respectively, and when computing probabilities from pheromone values, a weighted mean using the weights  $w_1$ ,  $w_2$  and  $w_3$  is applied.

The visibility function  $\eta(i, j)$  indicating how desirable it is to visit node  $v_j$  after node  $v_i$ , deserves special attention, since it must be defined in a problem-specific way. It can be composed as the sum of three terms  $\eta_1(i, j)$ ,  $\eta_2(i, j)$  and  $\eta_3(i, j)$ , which refer to the three objective functions:

*Term 1:* A rough estimation how many stops should lie on a “good” tour is required for determining  $\eta_1(i, j)$ . Let  $\bar{k}$  be the current estimate for the best number of

stops. We initialize  $\bar{k}$  by a constant, and update its value during program execution by setting  $\bar{k}$  equal to the average length of the tours contained in  $\Pi$  whenever the last set has changed. Based on  $\bar{k}$ , the visibility  $\eta_1(i, j)$  corresponding to objective function 1 is determined as follows:

$$\eta_1(i, j) = K_1 \cdot \begin{cases} c_{ij}^{-\alpha}, & \text{if } j \neq 1 \text{ or less than } \bar{k} \text{ nodes have been visited,} \\ \bar{c}, & \text{if } j = 1 \text{ and } \bar{k} \text{ or more nodes have been visited.} \end{cases}$$

In this formula,  $\alpha > 0$  is a constant parameter, and  $\bar{c}$  is another constant that is large compared to the values  $c_{ij}^{-q}$ . Hence, a return to the start node is made more probable when the estimated “good” tour length has already been reached.  $K_1$  is a constant calibration factor for the first term of the visibility function.

*Term 2:* While an ant constructs tour  $\pi$ , it records the average distance of the entire population to the stops already fixed on this tour, and it computes by which value this average distance is reduced if a certain node  $v_j$  is chosen as the next stop. This difference, multiplied by a calibration factor  $K_2$ , is chosen as the visibility term  $\eta_2(i, j)$ .

*Term 3:* For each settlement  $v_j$ , the number  $\lambda_j$  of potential stops within a distance of not more than  $M$  kilometers from  $v_j$  is determined. Then, we set

$$\eta_3(i, j) = K_3 \cdot \begin{cases} p_j/\lambda_j, & \text{if } \lambda_j \geq 1, \\ \epsilon, & \text{if } \lambda_j = 0, \end{cases}$$

where  $\epsilon > 0$  is a small constant. Thereby, settlements with a large number of inhabitants as well as settlements from which only few potential stops can be reached within a distance of not more than  $M$  kilometers, are favored. Favoring settlements with  $\lambda_j = 0$  would not be advantageous, since they cannot be serviced within a distance of not more than  $M$  kilometers anyway.

The resulting tour is post-optimized by local search. For this purpose, we use the well known 2-opt-procedure (see [9]). During the post-optimization of the tour, the tour stops remain fixed. Therefore, this step can improve objective function (1) while leaving objective functions (2) and (3) unchanged.

In Fig. 1, we give a pseudocode formulation of the overall algorithm.

For judging the computational complexity of P-ACO/LR, we assume that an upper bound  $\Lambda$  on the length of the list  $\Pi$  is given, with the following effect: In the case that the list would exceed length  $\Lambda$  by the insertion of a new element, the new element is only inserted if at least one element of  $\Pi$  can be deleted from the list by

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**Procedure P-ACO/LR**

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initialize pheromone matrices  $\tau_m = (\tau_m(i, j))$  ( $m = 1, 2, 3$ );
initialize output set  $\Pi$  as empty set;
for period  $q = 1$  to  $Q$  {
  draw weights  $w_1, w_2$  and  $w_3$  randomly from  $[0, 1]$ ;
  normalize the weights to  $w_1 + w_2 + w_3 = 1$ ;
  compute current weighted pheromone matrix as  $\tau = \sum_{m=1}^3 w_m \tau_m$ ;
  for round  $r = 1$  to  $R$  {
    for ant  $\sigma = 1$  to  $s$  {
      set node index  $i = 1$ ;
      while (node 1 not chosen for the second time) {
        compute visibility matrices  $\eta_m = (\eta_m(i, j))$  ( $m = 1, 2, 3$ );
        compute current weighted visibility matrix as  $\eta = \sum_{m=1}^3 w_m \eta_m$ ;
        compute transition probabilities  $p(i, j)$  to feasible successor nodes by (15);
        choose successor node  $j$  according to  $p(i, j)$ ;
        set  $i = j$  and mark  $i$  as already visited;
      } /* end while */
      improve the tour by 2-opt;
      store the tour;
    } /* end for ant */
    compute costs  $Z_m(\sigma)$  ( $m = 1, 2, 3; \sigma = 1, \dots, s$ ) for each of the found tours;
    compute current weighted costs  $Z(\sigma) = \sum_{m=1}^3 w_m Z_m(\sigma)$  ( $\sigma = 1, \dots, s$ );
    compute the best  $s_0$  tours  $\pi_1^*, \dots, \pi_{s_0}^*$  with respect to the values  $Z(\sigma)$ ;
    if (best tour  $\pi_1^*$  is non-dominated by all tours in  $\Pi$ )
      add  $\pi_1^*$  to  $\Pi$  and delete all tours from  $\Pi$  that are dominated by  $\pi_1^*$ ;
    for  $m = 1$  to  $3$ 
      do rank-based update for  $\tau_m$ , using  $\pi_1^*, \dots, \pi_{s_0}^*$  and  $\delta_0 = \text{const} \cdot w_m$ ;
      update  $\tau$  by setting  $\tau = \sum_{m=1}^3 w_m \tau_m$ ;
    } /* end for round */
  } /* end for period */
```

---

Fig. 1. Pseudocode P-ACO for Location-Routing.

domination. Furthermore, it is assumed that the number of performed 2-opt moves is bounded by a constant  $\omega$ , and that  $s_0$  is a (small) constant of order  $O(1)$ .

**Proposition 3.1.** On the assumptions indicated above, the worst-case complexity of P-ACO/LR is of order

$$O(QR [s |W| |V| + s \omega |V|^2 + \Lambda]).$$

**Proof.** In each of the  $R$  rounds of each of the  $Q$  periods, the actions contributing to the complexity in an essential way are the following:

- For determining  $Z_2$  for each of the  $s$  tours, the minima  $d(i, \pi)$  of  $O(|V|)$  distances have to be computed for  $|W|$  population centers, which yields an effort of  $O(s |V| |W|)$  in total. Note that  $Z_3$  can be computed in an analogous manner, replacing  $d(i, \pi)$  by  $\bar{d}(i, \pi) = 1$  if  $d(i, \pi) > M$  and  $\bar{d}(i, \pi) = 0$  otherwise, and that the computation of  $Z_1$  for the  $s$  tours requires only an effort of  $O(s |V|)$ .
- Applying a single 2-opt move to a tour of length  $O(|V|)$  requires a search among  $O(|V|^2)$  neighbor solutions; the changed objective function value for each neighbor solution can be determined in  $O(1)$  time. This yields a total effort of  $O(s \omega |V|^2)$  for the (up to)  $\omega$  2-opt moves.
- The three-dimensional cost vector of  $\pi_1^*$  has to be compared with up to  $\Lambda$  cost vectors of the elements in the list  $\Pi$ , and each of these elements may need to be deleted from the list (if dominated by  $\pi_1^*$ ), which can be done in  $O(1)$  time for a single deletion. This yields a total effort of  $O(\Lambda)$ .

□

## 3.2 Approach 2: Location by Multiobjective GA with Routing as Subprocedure

In our second approach, we deal with the two aspects location and routing separately from each other: In a *master procedure*, subsets  $V_0 \subseteq V$  of tour stops are selected from the set  $V$  of possible stops. For each subset  $V_0$  of this type, a *slave procedure* finds a short feasible tour  $\pi$  containing all nodes of  $V_0$  by solving the corresponding travelling salesperson problem (TSP) heuristically. For the obtained tour  $\pi$ , the three objective functions  $Z_1$ ,  $Z_2$  and  $Z_3$  are computed as described in section 2. Thus, we obtain a (three-criteria) evaluation of the current  $V_0$ .

In our implementation of the slave procedure, we have decided to apply simple constructive TSP heuristics followed by post-optimization using the 2-opt procedure

mentioned above, as a fast technique to get a heuristic solution of the TSP. For the constructive heuristics producing the initial solutions, we performed tests with the well-known Nearest Neighbor (NN) heuristic and with the Farthest Insertion (FI) heuristic (see Johnson and McGeoch [32]). Although FI is known to perform better than NN in general, it turned out that the post-optimization by 2-opt (which consumes the major part of the runtime, see the complexity result in Proposition 3.2) annihilated this advantage in our tests: there was no statistically significant difference in solution quality after 2-opt between the two approaches for computing initial solutions. This does not hold anymore for initial tours chosen *randomly*; in this case, we obtained poorer results. The experimental reports in Section 4 refer to the NN variant.

The more difficult question is how the master procedure should be realized. Since its task is to solve a multicriteria problem, P-ACO could be used again. For the master procedure, however, the structure of the feasible solutions is much simpler than in the combined location-routing setting of the previous subsection: now feasible solutions are subsets, representable by binary strings. Therefore, it is tempting to apply a multiobjective variant of a *Genetic Algorithm* (GA), a metaheuristic for which binary string representations of solutions are particularly natural.

There are several variants of multiobjective GA techniques; for a comprehensive survey, we refer the reader to Coello et al. [8]. In our problem context, we experimented with two variants: (i) the *Vector Evaluated Genetic Algorithm* (VEGA) by Schaffer [44], and (ii) the *Multi-Objective Genetic Algorithm* (MOGA) by Fonseca and Fleming [19].

### 3.2.1 Realization of the Master Procedure by VEGA

First of all, the master procedure generates an initial population of  $P$  subsets  $V_0$ , represented by binary strings. For each of these  $P$  solutions, the number of tour stops (i.e., the cardinality  $|V_0|$  of  $V_0$ ) is determined by drawing a random number uniformly distributed between  $\xi_{min} \cdot |V|$  and  $\xi_{max} \cdot |V|$ , where  $|V|$  is the number of potential stops, and  $\xi_{min} \in [0, 1]$  and  $\xi_{max} \in [0, 1]$  ( $\xi_{min} < \xi_{max}$ ) are minimum and maximum values for the share of selected stops, respectively. The two parameters  $\xi_{min}$  and  $\xi_{max}$  are determined in advance by educated guesses. After fixing  $|V_0|$  in this way, a specific subset  $V_0$  with given cardinality is chosen uniformly at random from the set of all  $\binom{|V|}{|V_0|}$  candidates. For each generated string, the slave procedure is called to compute an assigned tour  $\pi$ . This produces  $P$  tours.

Now, according to Schaffer's VEGA approach, the whole population is split into three fractions, each containing  $P/3$  tours, corresponding to the three objective functions  $Z_1$ ,  $Z_2$  and  $Z_3$ . From fraction  $m$ , a new set of  $P/3$  elements is selected

randomly by the standard selection mechanism of GA, according to objective  $Z_m$  ( $m = 1, 2, 3$ ). All the selected elements are then shuffled together to obtain a new population of total size  $P$ . To the elements (tours) of the new population, two types of *mutation* operators and a *crossover* operator are applied: The first mutation operator consists in a random 2-opt-move; the second mutation operator replaces a chosen tour stop by a currently not chosen tour stop and applies then the 2-opt procedure to improve the resulting tour. As a crossover operator, we apply the OX crossover for TSP tours proposed by Davis [11]. This is a two-point crossover where, from two parent tours, the medium parts are copied to the two offspring tours unchanged, while the other parts are copied in switched order, omitting already occurring symbols.

The population generation step described above is iterated  $R$  times. (Since these iterations play a similar role as the rounds in P-ACO/LR, we denote their number by the same symbol  $R$ .) Fig. 2 shows the resulting overall pseudocode for the master procedure.

**Proposition 3.2.** On the assumptions of Proposition 3.1 and with the same notation, the worst-case complexity of VEGA/LR including the execution of the slave procedure is of order

$$O(PR[|W||V| + \omega|V|^2 + \Lambda]).$$

**Proof.** The essential contribution to the complexity of the slave procedure comes from the 2-opt moves, hence the total effort for one execution of this procedure is of order  $O(\omega|V|^2)$  (see the proof of Proposition 3.1).

In the “repeat” loop, the slave procedure is called  $P$  times, which yields a total effort of  $O(P\omega|V|^2)$ .

In each of the  $R$  iterations of the “for generation...” loop, the actions contributing to the complexity in an essential way are the following:

- In the selection step, the determination of  $Z_1$  to  $Z_3$  for each element of  $Pop$  requires an effort of  $O(|V||W|)$ , which yields an effort of  $O(P|V||W|)$  in total.
- In mutation 2, the 2-opt procedure applied to each mutated element requires an effort of  $O(\omega|V|^2)$ , which yields an effort of  $O(P\omega|V|^2)$  in total.
- For each element of  $Pop$ , comparison with and eventual deletion of an element of the list  $\Pi$  requires an effort of  $O(\Lambda)$ , which yields an effort of  $O(P\Lambda)$  in total.

As it can be seen, the effort for the “repeat” loop is negligible, compared to that for the “for generation...” loop. □



---

**Procedure VEGA/LR**

---

```
repeat  $P$  times {
  choose cardinality  $|V_0|$  of subset  $V_0 \subseteq V$  at random;
  choose  $V_0 \subseteq V$  with given cardinality at random;
  call slave procedure to assign a tour  $\pi$  to  $V_0$ ;
  add  $\pi$  to initial population  $Pop$ ;
}
initialize output set  $\Pi$  as empty list;
for generation  $r = 1$  to  $R$  {
  split  $Pop$  into fractions  $Pop_1, Pop_2$  and  $Pop_3$ ;
  for  $m = 1$  to  $3$  {
    evaluate the elements of  $Pop_m$  according to  $Z_m$ ;
    select new fraction  $Pop_m^{new}$  from  $Pop_m$  according to standard GA rule;
  }
  establish new  $Pop$  by shuffling together  $Pop_1^{new}, Pop_2^{new}$  and  $Pop_3^{new}$ ;
  apply mutation 1 (2-opt move) to a certain fraction of  $Pop$ ;
  apply mutation 2 (tour stop exchange plus 2-opt procedure) to a certain
  fraction of  $Pop$ ;
  apply OX crossover to a certain percentage of pairs of elements of  $Pop$ ;
  for each element of  $Pop$ 
    if element is non-dominated by elements in  $\Pi$ , add it to  $\Pi$  and delete
    all dominated elements from  $\Pi$ ;
} /* end for  $r$  */
```

---

*Fig. 2.* Pseudocode VEGA for Location-Routing (Master Procedure).

### 3.2.2 Realization of the Master Procedure by MOGA

MOGA is similar to VEGA, so we keep our presentation short (for more details on the basic MOGA algorithm, the reader is referred to [19]) and concentrate on the differences. The main difference is that by a specific ranking mechanism, MOGA tries to avoid “speciation” effects, i.e., the evolution of “species” that excel only on a single objective function, but are poor with respect to the others.

The ranking is done as follows: Let us consider an individual  $\pi$  (in our case: a tour) in generation  $r$  of the population. It is counted how many other individuals of the current generation *dominate*  $\pi$  in the sense of the definition given at the beginning of section 3. Let  $\chi(\pi, m)$  denote the number of elements dominating  $\pi$ . Then the rank of  $\pi$  in the current generation is defined as

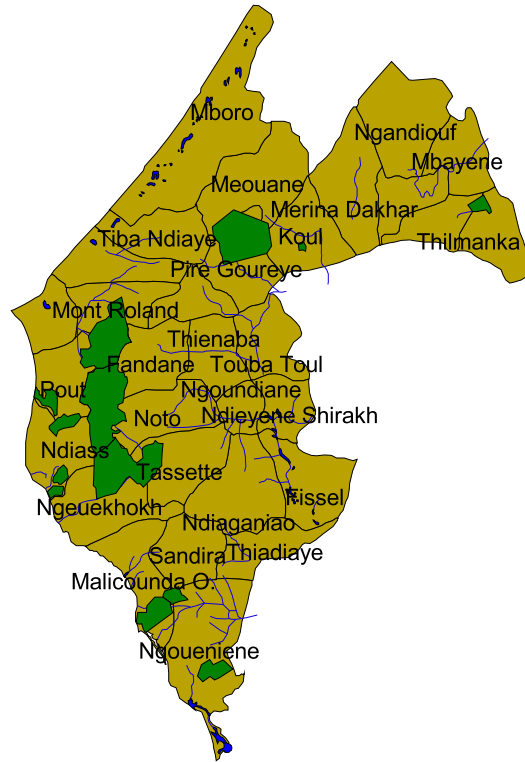
$$\text{rank}(\pi, m) = 1 + \chi(\pi, m).$$

The lower the rank, the better is the given individual compared to the other individuals in the current generation. Now, instead of partitioning the population into fractions and performing selection in each fraction according to the corresponding objective function as in VEGA, the MOGA approach performs selection from the entire population based on fitness values that are linear functions of the ranks. In this way, good compromise solutions, which are unable to survive under the VEGA scheme, are encouraged.

In order to avoid premature convergence, MOGA is supplemented by a *niche-formation* technique trying to distribute the population over the Pareto-efficient region. We omit the details and refer the reader to the literature cited above.

## 4 Example Study: Tour Planning for the Thiès Region in Senegal

Senegal is a country at the West-African coast with a total population of about 9 600 000 (WHO [31]). The country became independent from the former colonial power France in 1960. Senegal is largely muslim, politically stable, and a multi-party democracy. It is a small, poor and moderate secular nation, but still among the world’s least developed countries, despite the relatively modern capital city. Although Senegal’s economic growth has recently improved, it has been virtually negated by high population growth of 2.5%. The per capita Gross Domestic Product (GDP) is US\$ 500, of which 4.7% is spent on healthcare (WHO [31], USAID [30]).



*Fig. 3.* Map of the Thiès region in Senegal.

Social services in Senegal remain extremely limited, especially for women and children. Mature mortality is high (around 30 % between 15 and 59), and many infants die of preventable diseases. Senegal suffers from relatively high childhood mortality rates (14 % per 1 000 dying under age of 5 years), and women still bear on average 5.2 children each. Live expectancy at birth is 55.8 years, and the expectancy of lost healthy years at birth due to poor health is around 10 years.

The road net comprised around 14 500 km in 1998, and the railway network around 1 257 km, which is comparably few for a country of 197 000 square km. Thus, finally, a majority of the Senegalese population does not have access to adequate health services and almost half do not have access to safe water (US-Embassy Dakar [29]).

For testing our approach, we chose the Thiès region in the west part of the country. Fig. 3 shows a map of this region. It is divided into three departments (Mbour, Thiès and Tivaouane), each consisting of some smaller districts (“arrondissements”).

Using ArcView version 3.2, we built a GIS (geoinformation system) providing the essential information for our purpose. In particular, we recorded 500 settlements (from cities to small villages) with their numbers of inhabitants, as well as the road connections between these settlements. The GIS information was obtained

based on detailed regional maps and on census data from 1988. We remark that more recent data exist, but are not yet released for public use by the government. For the purpose of testing the computational approaches, it can be expected that inserting the new data will not lead to essentially different results. Location data on settlements and roads were digitized semi-mechanically in ArcView. Distances were then obtained electronically, and the roads were classified into five classes, for which average velocities of cars were estimated.

Our aim was to compute plans covering the entire region by a small number of tours, each of which should have a period  $T$  of one week, such that medical service could be provided weekly. Already existing medical facilities, both stationary and mobile, were disregarded (see the remarks at the beginning of section 2). A rough calculation showed that (at least) four tours were required to achieve the indicated goal, keeping our three cost functions below reasonable aspiration levels. We chose the following thresholds as aspiration levels: Objective  $Z_1$  (nonproductive share of work): 0.90; objective  $Z_2$  (average walking distance): 3 km; objective  $Z_3$  (share of uncovered population): 0.4.

For clustering the settlements to be served by each of the four tours, we decided to respect the partition of the region into districts and comprised these districts to four *service areas* “north” (N), “north mid” (NM), “mid south” (MS) and “south” (S), and in such a way that the numbers of settlements were approximately equally distributed. Fig. 4 shows the road network and the resulting area partition.

The problem described in section 2 was solved by each of the three approaches P-ACO, VEGA and MOGA for each area separately. The parameters of our model have been chosen as follows:

- Total working time of a member of the MF personnel during the period:  $T = 40$  hours, i.e., one week.
- Time for the setup of the MF at a stop per member of the MF personnel:  $\mu = 1/6$  hours, i.e., 10 minutes.
- Upper bound for an acceptable walking distance to the nearest tour stop:  $M = 8$  km.

The heuristic approaches were implemented on a PC Pentium III, 1400 Mhz, under operating system Windows XP. We have chosen the following parameter values for the three algorithms: (i) P-ACO/LR:  $s = 20$ ,  $s_0 = 3$ ,  $Q = 30$ ,  $R = 30$ . (ii) VEGA/LR:  $P = 100$ ,  $R = 600$ . (iii) MOGA/LR:  $P = 100$ ,  $R = 600$ . Programs were written in C++ (compiler: Borland C++, version 6). Runtimes varied according to the number of non-dominated solutions produced in each single case.

In the average, a run of P-ACO required about 12 minutes, whereas VEGA and MOGA produced the result set already within about 3 minutes in the average.

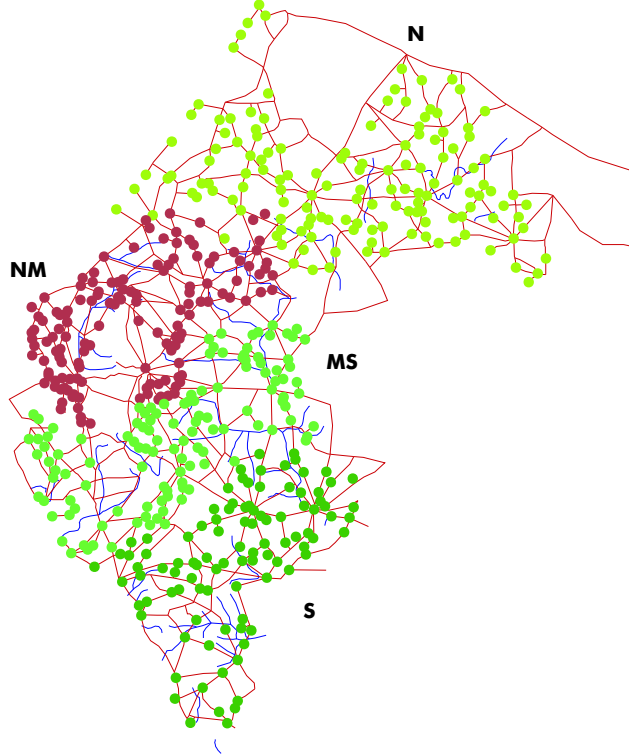


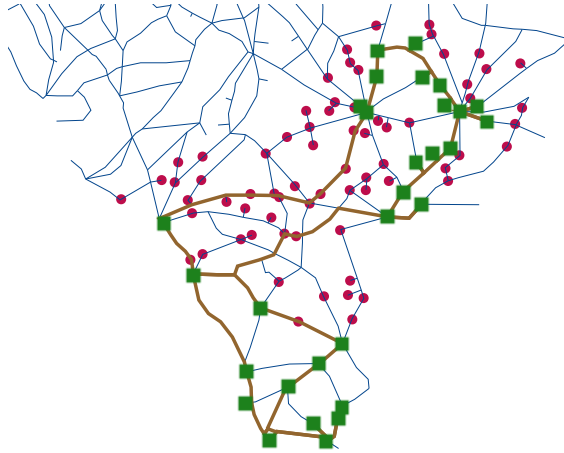
Fig. 4. Road network and chosen area partition.

Before presenting the overall comparison between the three approaches, let us look at some typical solutions for the “south” area, which has 99 potential tour stops. Fig. 5 shows a tour produced by P-ACO with cost values  $Z_1 = 0.48$ ,  $Z_2 = 2.2$  and  $Z_3 = 0.14$ . The tour contains 32 stops (squares in the figure). As it can be seen, this is a solution where objective (1) has been favored: A degree of 52 % of medically productive working time is reached, which is rather high, compared to other obtained solutions. The price for this favorable value is a relatively high average distance to tour stops (2.2 km), and a poor coverage rate: 14 % of all people do not reach a tour stop within a distance of 8 km.

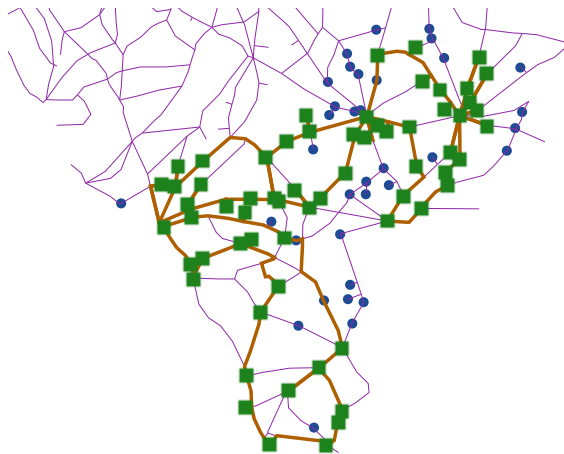
In Fig. 6 – also this solution has been produced by P-ACO – a tour is presented where objective (3) has been favored. The corresponding cost values are  $Z_1 = 0.90$ ,  $Z_2 = 0.76$  and  $Z_3 = 0.04$ . The tour has 54 stops. Now, both coverage rate and average distance are favorable, but the economic efficiency is very poor: only 10 % of the working time is spent on medical treatment. Obviously, this is an extreme solution that will most probably not be chosen by the decision makers.

Fig. 7 shows a tour produced by MOGA, with cost values  $Z_1 = 0.50$ ,  $Z_2 = 2.3$  and  $Z_3 = 0.12$ . The achieved objective function values resemble those of the tour shown in Fig. 5.

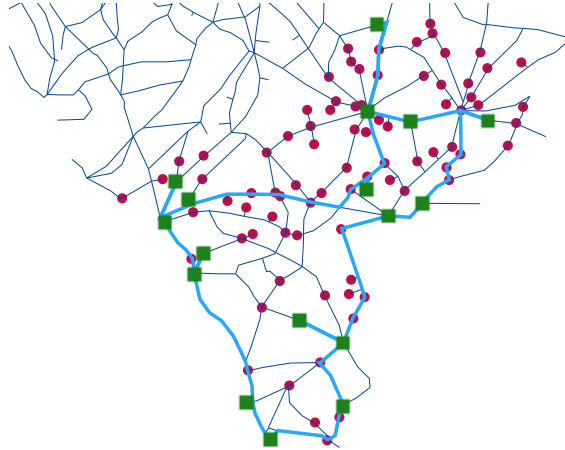
The three shown solutions have some tour stops in common; in particular, Mbour, a larger city in the west of the area “south”, gets a tour stop in each of the three cases, which is evidently reasonable.



*Fig. 5.* P-ACO example tour (1): good objective function value with respect to objective 1.



*Fig. 6.* P-ACO example tour (2): good objective function value with respect to objective 3.



*Fig. 7.* MOGA example tour: good objective function value with respect to objective 1.

Tables 1 – 4 (corresponding to the areas “south”, “mid south”, “north mid” and “north”, respectively) present overall comparison results for the implemented techniques P-ACO, VEGA and MOGA. Comparisons are performed pairwise (P-ACO vs. MOGA, P-ACO vs. VEGA, and VEGA vs. MOGA). The algorithm indicated in a line is taken as the “first” algorithm of the comparison, the algorithm indicated in a column is taken as the “second” algorithm of the comparison. In each line, after the name of the algorithm, we indicate in dots the number of non-dominated solutions produced by this algorithm. The entries of the table are pairs of integers which are to be interpreted as follows:

- First integer: number of solutions produced by the algorithm indicated in the line that are dominated by some solution in the output set of the algorithm indicated in the column.
- Second integer: number of solutions produced by the algorithm indicated in the line that are *not* dominated by any solution of the algorithm indicated in the column.

Thus, a final judgement on the relative merits of two algorithms can be made by looking at the ratio of the second integers in the two entries referring to the comparison of these two algorithms. For instance, when P-ACO is compared to MOGA on the data of the “south” area, these integers are 25 for P-ACO and 8 for MOGA. This means that if the solutions of P-ACO and those of MOGA are composed to a joint solution set (containing 80 solutions), and if dominated solutions are eliminated afterwards, then there remains a resulting set of 33 solutions, 25 of which

have been contributed by P-ACO, and only 8 of which have been contributed by MOGA.

Interpreting the results in the four tables, we see that for area “south”, P-ACO and VEGA performed equally good, and MOGA performed worse. For “mid south”, MOGA performed best and VEGA performed worst. For “north mid”, all the three approaches performed nearly equally good, with a slight advantage for MOGA over VEGA. For “north”, the ranking was similar as for “mid south”. Thus, there is no consistent winner, but we can state that MOGA was the only approach that eventually was distinctly better than both others, and P-ACO was the only approach that never was *worse* than both others.

A finer analysis reveals that MOGA solution sets typically differ from P-ACO solution sets in the following aspect: MOGA usually produces a larger variety of sometimes extreme solution candidates (VEGA tends even more to the extremes), whereas P-ACO seems to put more emphasis on good compromise solutions. Since both properties have their particular merits, it can be suggested to use, in a decision support system, both the P-ACO and the MOGA approach and to combine the solution sets, omitting dominated solutions.

Algorithm (# solutions)	Dominated sol. / Remaining sol.		
	P-ACO	MOGA	VEGA
P-ACO (25)	—	0 / 25	0 / 25
MOGA (55)	47 / 8	—	44 / 11
VEGA (36)	12 / 24	7 / 29	—

Table 1. Comparative results for area “south”.

Algorithm (# solutions)	Dominated sol. / Remaining sol.		
	P-ACO	MOGA	VEGA
P-ACO (26)	—	8 / 18	6 / 20
MOGA (42)	1 / 41	—	1 / 41
VEGA (17)	2 / 15	16 / 1	—

Table 2. Comparative results for area “mid south”.

Algorithm (# solutions)	Dominated sol. / Remaining sol.		
	P-ACO	MOGA	VEGA
P-ACO (79)	—	18 / 61	15 / 64
MOGA (108)	49 / 59	—	32 / 76
VEGA (94)	29 / 65	42 / 52	—



Table 3. Comparative results for area “north mid”.

Algorithm (# solutions)	Dominated sol. / Remaining sol.		
	P-ACO	MOGA	VEGA
P-ACO (65)	—	15 / 50	8 / 57
MOGA (75)	17 / 58	—	4 / 71
VEGA (58)	27 / 31	53 / 5	—

Table 4. Comparative results for area “north”.

## 5 Conclusions

We have given a multiobjective combinatorial optimization (MOCO) formulation for a location-routing problem in healthcare management: For a mobile healthcare facility, a closed tour on a suitably selected subset of a given set of population nodes has to be found. Tours are evaluated according to the following three criteria: (i) an economic efficiency criterion that can be expressed as a weighted average of the number of tour stops and the tour length, (ii) the p-median criterion of average distances to the nearest tour stops, and (iii) a coverage criterion that measures the percentage of the population unable to reach a tour stop within a predefined maximum distance.

Three algorithms to compute approximations to the set of Pareto-efficient solutions of the described MOCO problem have been developed. The first uses the P-ACO technique and performs selection of tour stops and tour construction simultaneously, whereas the second and the third use the VEGA and the MOGA variant, respectively, of Genetic Algorithm approaches to MOCO, and perform the selection of tour stops on an upper procedure level and tour construction in a 2-opt subprocedure.

Our computational experiments were carried out for the Thiès region in Senegal, which we partitioned into four sub-areas. For each of these areas, sets of Pareto-efficient tours were computed by the three algorithmic approaches. From the comparative evaluation of the solutions, we are inclined to suggest a combination of P-ACO and MOGA as a promising technique for providing the political decision maker with an adequate set of good solution candidates. Further experiments will be required to get more information on the relative merits of the investigated approaches.

The focus of this paper is on problem formulation and suitable optimization techniques. For test purposes, we have investigated the decision problem for the Thiès region on the simplifying assumption that *all* medical services are to be covered by the mobile facilities alone; existing fixed healthcare facilities have not been included into the consideration. For this reason, a formulation of concrete policy implications is outside the scope of this study. Nevertheless, some cautious conclusions can already be drawn. The results in Section 4 show that on the indicated assumptions, even in the absence of healthcare facilities in the considered region, four mobile units could provide access to medical service for about 85 % of the population within 8 km distance, with an average distance of about 2 km. The overhead of personnel costs for non-medical activities would be about 100 % in this case. Of course, the made assumptions will have to be discussed, but we feel that the results indicate at least that supplementing locally fixed medical stations by mobile healthcare units should be taken into consideration as a possibly useful measure when an extension of access to medical service in a country with a low healthcare budget is intended. The advantages of mobile units will have to be traded off against evident disadvantages, for example, the lack of *continuous* care for patients by medical personnel. It should be kept in mind, however, that by a suitable mix between the “stationary” and the “mobile” policy, the system can be fine-tuned to the particular needs of a concrete country. It may be preferable to supplement the stationary hospitals or healthcare stations by some few mobile units providing care even at a rather low level instead of excluding large parts of the population totally from medical supply.

Future research should address also some important extensions of the model itself. We give a few examples. First, of course, the just-mentioned assumption should be dropped that medical supply in the considered area is provided exclusively by the mobile healthcare units. Instead, existing (or planned) stationary facilities should be taken into account as well. In a first approximation, this extension is very easy to perform: All that needs to be done is to reduce, in the problem instance description, the numbers of inhabitants of each settlement by the (estimated) numbers of people who are already supplied by stationary healthcare facilities such as hospitals or primary healthcare dispensaries (located close enough to the settlement). However, what would not be obtained in this way are *overall* objective function values for average accessibility and coverage, including the services of the fixed facilities. Therefore, it would be desirable to include the supply delivered by the fixed facilities explicitly into the model, which requires an essential (although rather straightforward) generalization of the model formulation.

Second, results of studies on the effect of the distance from home to health facilities (cf. Morris and Lee [37], Müller et al. [38]) should be used to refine the model.

Third, an *availability* criterion (influenced by opening times, relation between demand and supply, queues, types of diseases, medicaments and doctors) should enter into the model as a fourth objective. In our model in its present form, the overall duration of the tour is fixed in advance, and the distribution of the durations of stay of the mobile facility to the single tour stops is left open. Evidently, duration of stay has significant influence on the quality of service, but this question is confronted with a tradeoff: Prolonged stays facilitate the reduction of queues and improve service, but on the other hand, they also increase the overall time for one cycle of the tour, which means that the visits of the MF at a special tour stop occur less frequently. Thus, also durations should be considered as decision variables and be subject to optimization, preferably based on statistical data about demands and with the aid of an appropriate stochastic model.

Finally, the patient data should be classified according to the severity of their diseases. For example, a distinction between emergency and non-emergency cases could be carried out. In our basic framework, we assume that not only the chosen tour itself, but also the durations of stay are fixed in advance (otherwise, people could not rely on meeting the MF at a known date). However, it can make sense to admit exceptions, including the possibility of changing the tour, depending on the occurrence of emergency cases. This could lead to longer distances or waiting times for non-emergency patients, but save the lives of a number of emergency patients. Formally, tour planning with possible tour changes would require a stochastic-dynamic optimization model. Such optimization problems are notoriously hard to treat computationally; effort invested in a "on-line" decision support system enabling reactions on emergency cases may be well-spent nevertheless.

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