Chapter 7

Production, Capacity and Material Planning

Production, Capacity and Material Planning

- **Production plan**
  - quantities of final product, subassemblies, parts needed at distinct points in time
- **To generate the Production plan we need:**
  - end-product demand forecasts
  - Master production schedule

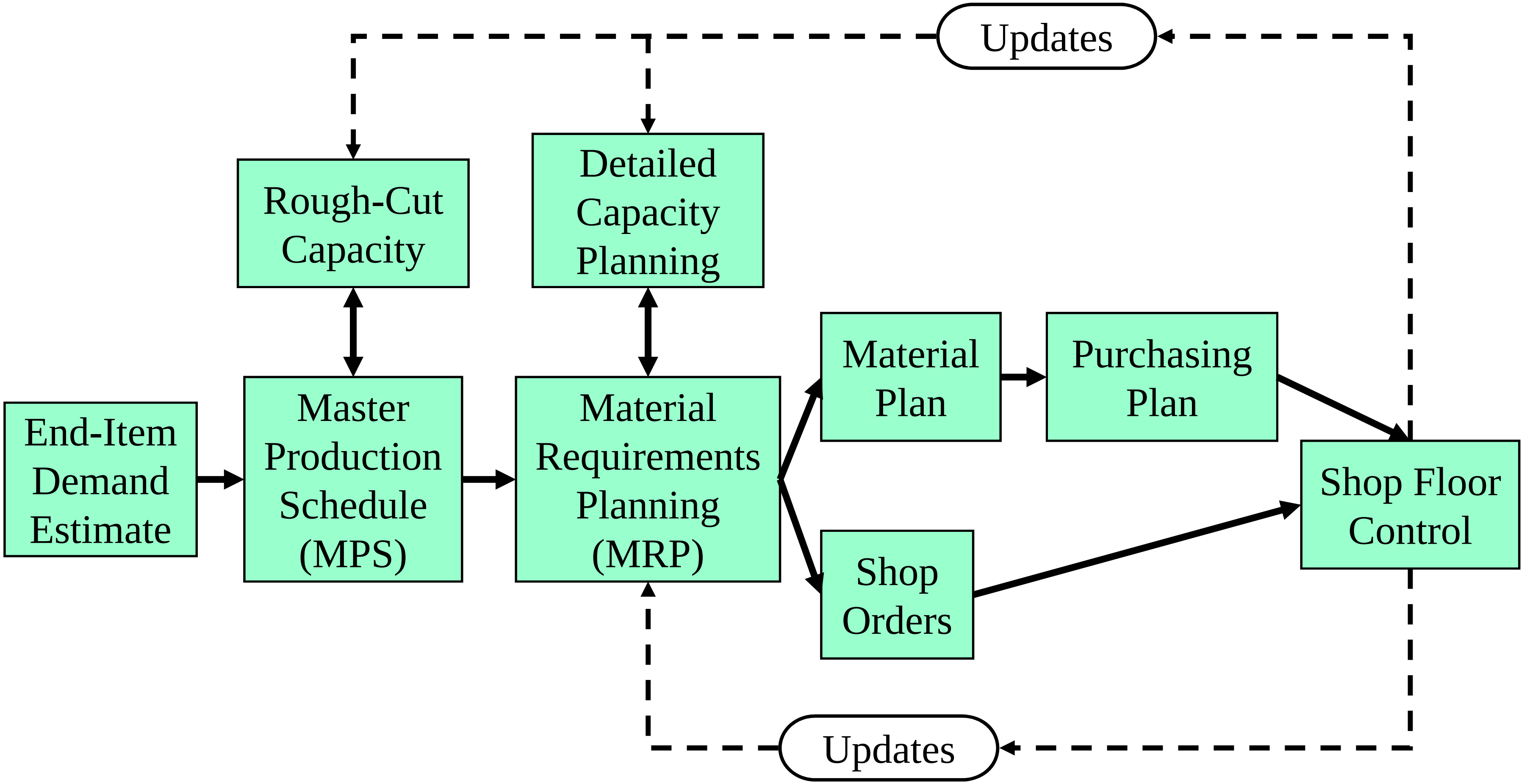
- **Master production schedule (MPS)**
  - delivery plan for the manufacturing organization
  - exact amounts and delivery timings for each end product
  - accounts for manufacturing constraints and final goods inventory

Production, Capacity and Material Planning

Based on the MPS:

- **rough-cut capacity planning**
- **Material requirements planning**
  - determines material requirements and timings for each phase of production
  - *detailed capacity planning*

Production, Capacity and Material Planning



Master Production Scheduling

- **Aggregate plan**
- **demand estimates for individual end-items**
- **demand estimates vs. MPS**
  - inventory
  - capacity constraints
  - availability of material
  - production lead time
  - ...
- **Market environments**
  - make-to-stock (MTS)
  - make-to-order (MTO)
  - assemble-to-order (ATO)

Master Production Scheduling

- **MTS**
  - produces in batches
  - minimizes customer delivery times at the expense of holding finished-goods inventory
  - MPS is performed at the end-item level
  - production starts before demand is known precisely
  - small number of end-items, large number of raw-material items
- **MTO**
  - no finished-goods inventory
  - customer orders are backlogged
  - MPS is order driven, consisits of firm delivery dates

Master Production Scheduling

- **ATO**
  - large number of end-items are assembled from a relatively small set of standard subassemblies, or modules
  - automobile industry
  - MPS governs production of modules (forecast driven)
  - Final Assembly Schedule (FAS) at the end-item level (order driven)
  - 2 lead times, for consumer orders only FAS lead time relevant

Master Production Scheduling

- MPS- SIBUL manufactures phones
  - **three desktop models A, B, C**
  - **one wall telephone D**
  - **MPS is equal to the demand forecast for each model**

WEEKLY MPS (= FORECAST)		**Jan**				**Feb**			
		Week				**Week**			
Product		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Model A		1000	1000	1000	1000	2000	2000	2000	2000
Model B			500	500		350			350
Model C		1500	1500	1500	1500	1000		1000	1000
Model D		600		600			300	200	
weekly total		3100	3000	3600	2500	3350	2300	3200	3350
monthly total		12200				12200			

Master Production Scheduling

- **MPS Planning - Example**
  - **MPS plan for model A of the previous example:**
  - **Make-to-stock environment**
  - **No safety-stock for end-items**
    - $I_t = I_{t-1} + Q_t - \max\{F_t, O_t\}$
    - I_t = end-item inventory at the end of week t
    - Q_t = manufactured quantity to be completed in week t
    - F_t = forecast for week t
    - O_t= customer orders to be delivered in week t

INITIAL DATA Model A		Jan				Feb			
Current Inventory = 1600		Week				Week			
		1	2	3	4	5	6	7	8
forecast Ft		1000	1000	1000	1000	2000	2000	2000	2000
orders Ot		1200	800	300	200	100			

Master Production Scheduling

- **Batch production: batch size = 2500**
  - I_t **= max{0,** I_{t-1} **} – max{**F_t**,** O_t**}**

$$Q_t = \begin{cases} 0, \text{if } I_t > 0 \\ 2500, \text{otherwise} \end{cases}$$

  - I_1 **= max{0, 1600} – max{1000, 1200} = 400 >0**
  - I_2 **= max{0, 400} – max{1000, 800} = -600 <0 => Q2 = 2500**
  - I_2 **= 2500 + 400 – max{1000, 800} = 1900, etc.**

MPS		**Jan**				**Feb**			
Current Inventory = 1600		**Week**				**Week**			
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
forecast Ft		1000	1000	1000	1000	2000	2000	2000	2000
orders Ot		1200	800	300	200	100			
Inventory It	1600	400	1900	900	2400	400	900	1400	1900
MPS Qt			**2500**		**2500**		**2500**	**2500**	**2500**
ATP		**400**	**1400**		**2200**		**2500**	**2500**	**2500**

Master Production Scheduling

- Available to Promise (ATP)
  - **ATP_1 = 1600 + 0 – 1200 = 400**
  - **ATP_2 = 2500 –(800 + 300) = 1400, etc.**

  - **Whenever a new order comes in, ATP must be updated**
- Lot-for-Lot production

MPS		**Jan**				**Feb**			
Current Inventory = 1600		**Week**				**Week**			
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
forecast Ft		1000	1000	1000	1000	2000	2000	2000	2000
orders Ot		1200	800	300	200	100			
Inventory It	1600	400	0	0	0	0	0	0	0
MPS Qt		**0**	**600**	**1000**	**1000**	**2000**	**2000**	**2000**	**2000**
ATP		**400**	**0**	**700**	**800**	**1900**	**2000**	**2000**	**2000**

Master Production Scheduling

- **MPS Modeling**
  - differs between MTS-ATO and MTO
  - find final assembly lot sizes
  - additional complexity because of joint capacity constraints
  - cannot be solved for each product independently

Master Production Scheduling

⌘ **Make-To-Stock-Modeling**

Q_{it} = production quantity of product i in period t

I_{it} = Inventory of product i at end of period t

D_{it} = demand (requirements) for product i in time period t

a_i = production hours per unit of product i

h_i = inventory holding cost per unit of product i per time period

A_i = set-up cost for product i

G_t = production hours available in period t

$y_{it} = 1$, if set-up for product i occurs in period t ($Q_{it} > 0$)

Master Production Scheduling

- **Make-To-Stock-Modeling**

$$\min \sum_{i=1}^{n} \sum_{t=1}^{T} \left(A_i y_{it} + h_i I_{it} \right)$$

$$I_{i,t\text{-}1} + Q_{it} - I_{it} = D_{it} \qquad \text{for all } (i,t)$$

$$\sum_{i=1}^{n} \mathrm{a_i} Q_{it} \le G_t \qquad \text{for all } t$$

$$\mathrm{Q_{it}} - y_{it} \sum_{k=1}^{T} D_{ik} \le 0 \qquad \text{for all } (i,t)$$

$$\mathrm{Q_{it}} \ge 0; I_{it} \ge 0; y_{it} \in \{0,1\}$$

Master Production Scheduling

- **Assemble-To-Order Modeling**
- **two master schedules**
  - MPS: forecast-driven
  - FAS: order driven
- **overage costs**
  - holding costs for modules and assembled products
- **shortage costs**
  - final product assemply based on available modules

    → no explicit but implicit shortage costs for modules
  - final products: lost sales, backorders

Master Production Scheduling

- *m* module types and *n* product types
- Q_{kt} = quantity of module k produced in period t
- g_{kj} = number of modules of type k required to assemble order j

- **Decision Variables:**
- I_{kt} = inventory of module k at the end of period t
- y_{jt} = 1, if order j is assembled and delivered in period t; 0, otherwise
- h_k = holding cost
- π_{jt} = penalty costs, if order j is satisfied in period t and order j is due in period t' (t'<t); holding costs if t' > t

Master Production Scheduling

- **Assemble-To-Order Modeling**

$$\min \sum_{k=1}^{m} \sum_{t=1}^{L} h_k I_{kt} + \sum_{j=1}^{n} \sum_{t=1}^{L} \pi_{jt} y_{jt}$$

subject to

$$I_{kt} = I_{k,t-1} + Q_{kt} - \sum_{j=1}^{n} g_{kj} y_{jt} \qquad \text{for all (k, t)}$$

$$\sum_{j=1}^{n} a_j y_{jt} \le G_t \qquad \text{for all t}$$

$$\sum_{t=1}^{L} y_{jt} = 1 \qquad \text{for all j}$$

$$I_{kt} \ge 0; \quad y_{jt} \in \{0,1\} \qquad \text{for all (j, k, t)}$$

Master Production Scheduling

- **Capacity Planning**
  - Bottleneck in production facilities
  - Rough-Cut Capacity Planning (RCCP) at MPS level
  - feasibility
  - detailed capacity planning (CRP) at MRP level
  - both RCCP and CRP are only providing information

Master Production Scheduling

MPS:	January			
	Week			
Product	1	2	3	4
A	1000	1000	1000	1000
B	-	500	500	-
C	1500	1500	1500	1500
D	600	-	600	-

	Bill of capacity (min)	
	Assembly	Inspection
A	20	2
B	24	2.5
C	22	2
D	25	2.4

	Capacity requires (hr)				
	Week				
	1	2	3	4	Available capacity per week
Assembly	1133	1083	1333!!	883	1200
Inspection	107	104	128!!	83	110

- **weekly capacity requirements?**
- **Assembly: 1000*20 + 1500*22 + 600*25 = 68000 min = 1133,33 hr**
- **Inspection: 1000*2 + 1500*2 + 600*2,4 = 6440 min = 107,33 hr etc.**
- **available capacity per week is 1200 hr for the assembly work center and 110 hours for the inspection station;**

Master Production Scheduling

- **Infinite capacity planning (information providing)**
- **finding a feasible cost optimal solution is a NP-hard problem**

- **if no detailed bill of capacity is available: capacity planning using overall factors (globale Belastungsfaktoren)**
  - required input:
  - MPS
  - standard hours of machines or direct labor required
  - historical data on individual shop workloads (%)

- Example from Günther/Tempelmeier
  - C133.3: overall factors

Master Production Scheduling

capacity planning using overall factors

	week					
product	**1**	**2**	**3**	**4**	**5**	**6**
A	100	80	120	100	120	60
B	40	-	60	-	40	-

product	**work on critical machine**	**work on non-critical machine**	**Total**
A	1	2	3
B	4	2	6

historic capacity requirements on critical machines:
40% on machine a
60% on machine b

Master Production Scheduling

in total 500 working units are available per week, 80 on machine a and 120 on machine b;

Solution:

overall factor = time per unit x historic capacity needs

product A:

machine a: 1 x 0,4 = 0,4

machine b: 1 x 0,6 = 0,6

product B:

machine a: 4 x 0,4 = 1,6

machine b: 4 x 0,6 = 2,4

Master Production Scheduling

capacity requirements: product A

machine				**week**		
	1	**2**	**3**	**4**	**5**	**6**
a	40	32	48	40	48	24
b	60	48	72	60	72	36
other	200	160	240	200	240	120

capacity requirements: product B

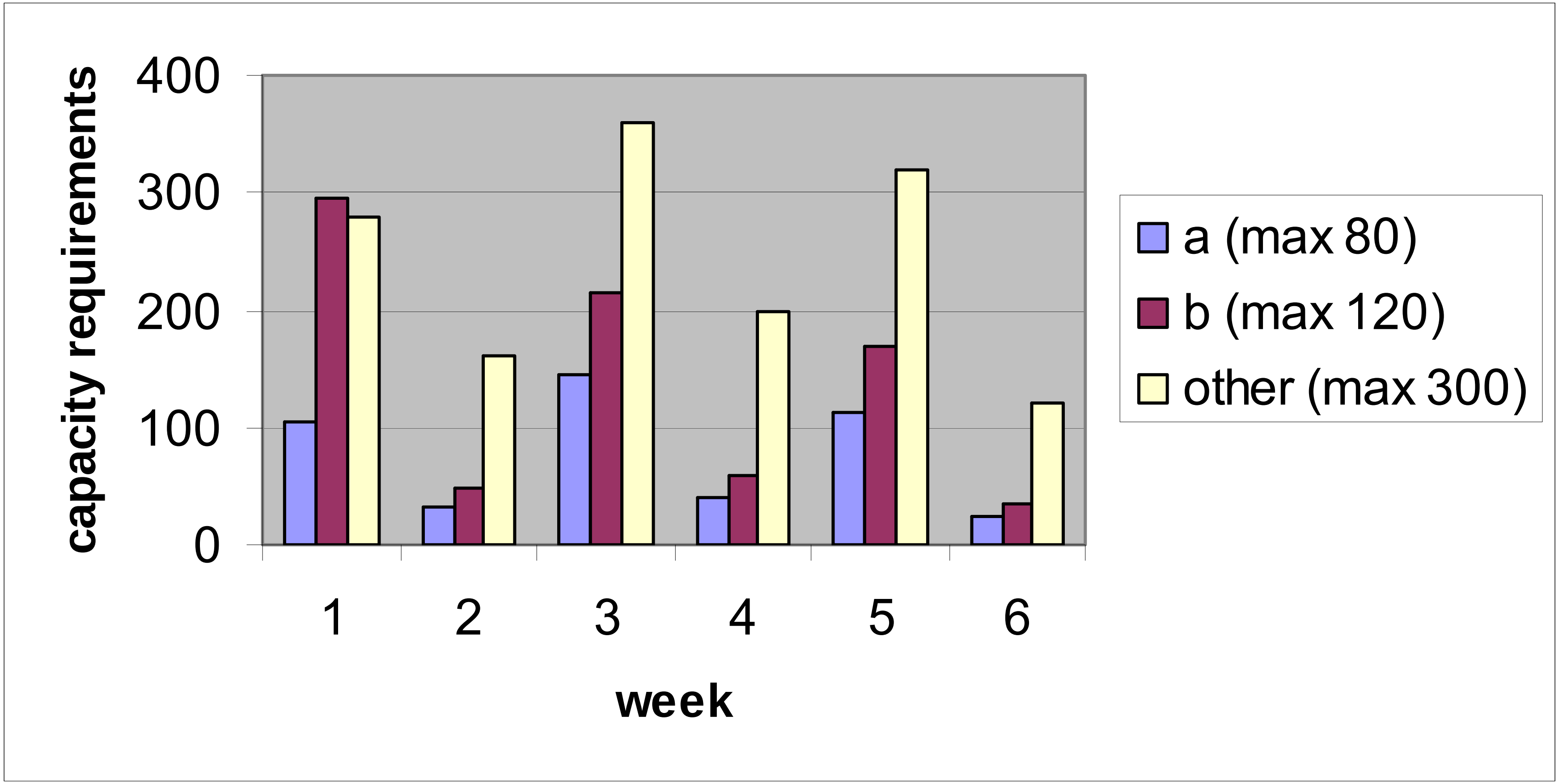
machine				**week**		
	1	**2**	**3**	**4**	**5**	**6**
a	64	-	96	-	64	-
b	96	-	144	-	96	-
other	80	-	120	-	80	-

Master Production Scheduling

total capacity requirements

machine			**week**			
	1	**2**	**3**	**4**	**5**	**6**
a	104	32	144	40	112	24
b	156	48	216	60	168	36
other	280	160	360	200	320	120

Master Production Scheduling



Master Production Scheduling

- **Capacity Modeling**
  - heuristic approach for finite-capacity-planning
  - based on input/output analysis
  - relationship between capacity and lead time

  - G= work center capacity
  - R_t= work released to the center in period t
  - Q_t= production (output) from the work center in period t
  - W_t= work in process in period t
  - U_t= queue at the work center measured at the beginning of period t, prior to the release of work
  - L_t= lead time at the work center in period t

Master Production Scheduling

$$Q_t = \min\{G, U_{t-1} + R_t\}$$

$$U_t = U_{t-1} + R_t - Q_t$$

$$W_t = U_{t-1} + R_t = U_t + Q_t$$

$$L_t = \frac{W_t}{G}$$

- **Lead time is not constant**
- **assumptions:**
  - constant production rate
  - any order released in this period is completed in this period

Master Production Scheduling

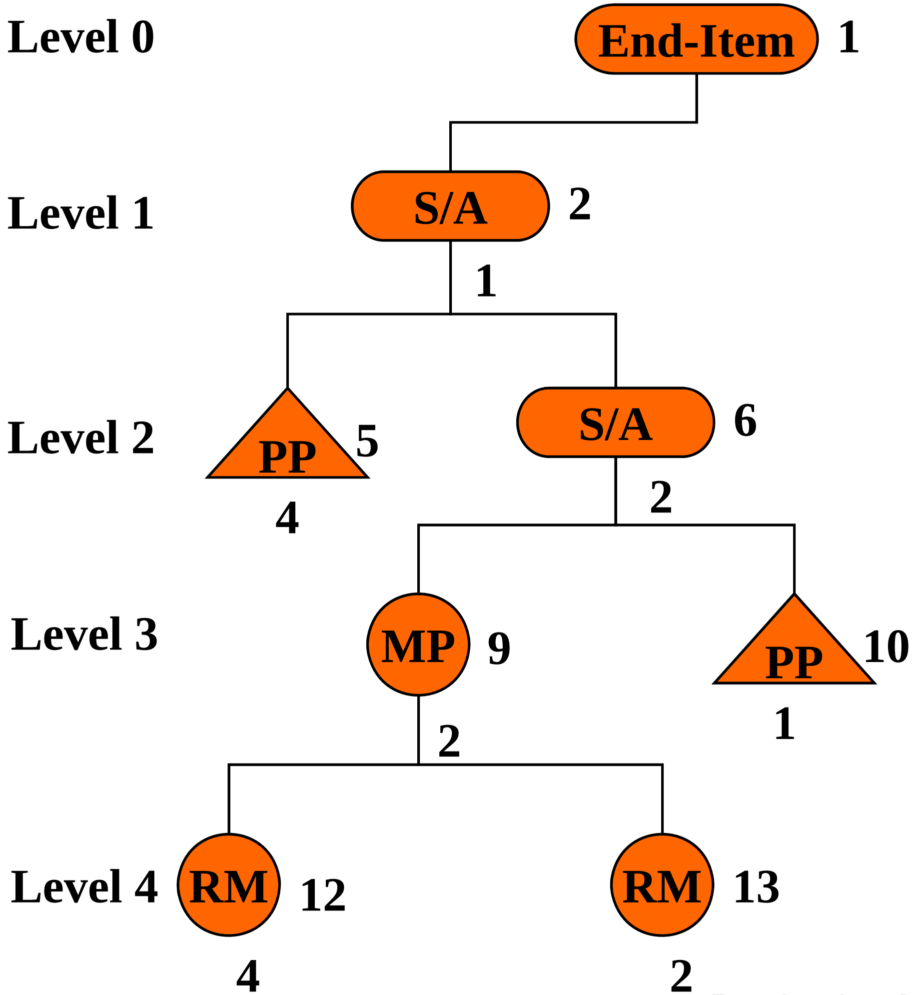
- **Example**

	Period						
	0	1	2	3	4	5	6
G (hr/week)		36	36	36	36	36	36
R_t (hours)		20	30	60	20	40	40
Q_t (hours)		*30*	*30*	*36*	*36*	*36*	*36*
U_t (hours)	10	*0*	*0*	*24*	*8*	*12*	*16*
W_t (hours)		*30*	*30*	*60*	*44*	*48*	*52*
L_t(weeks)		*0,83*	*0,83*	*1,67*	*1,22*	*1,33*	*1,44*

Material Requirements Planning

- **Inputs**
  - master production schedule
  - inventory status record
  - bill of material (BOM)
- **Outputs**
  - planned order releases
    - purchase orders(supply lead time)
    - workorders(manufacturing lead time)

Material Requirements Planning



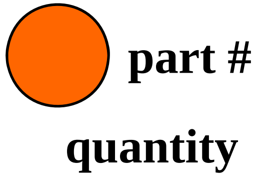
Legend:

S/A = subassembly

PP = purchased part

MP = manufactured part

RM = raw material

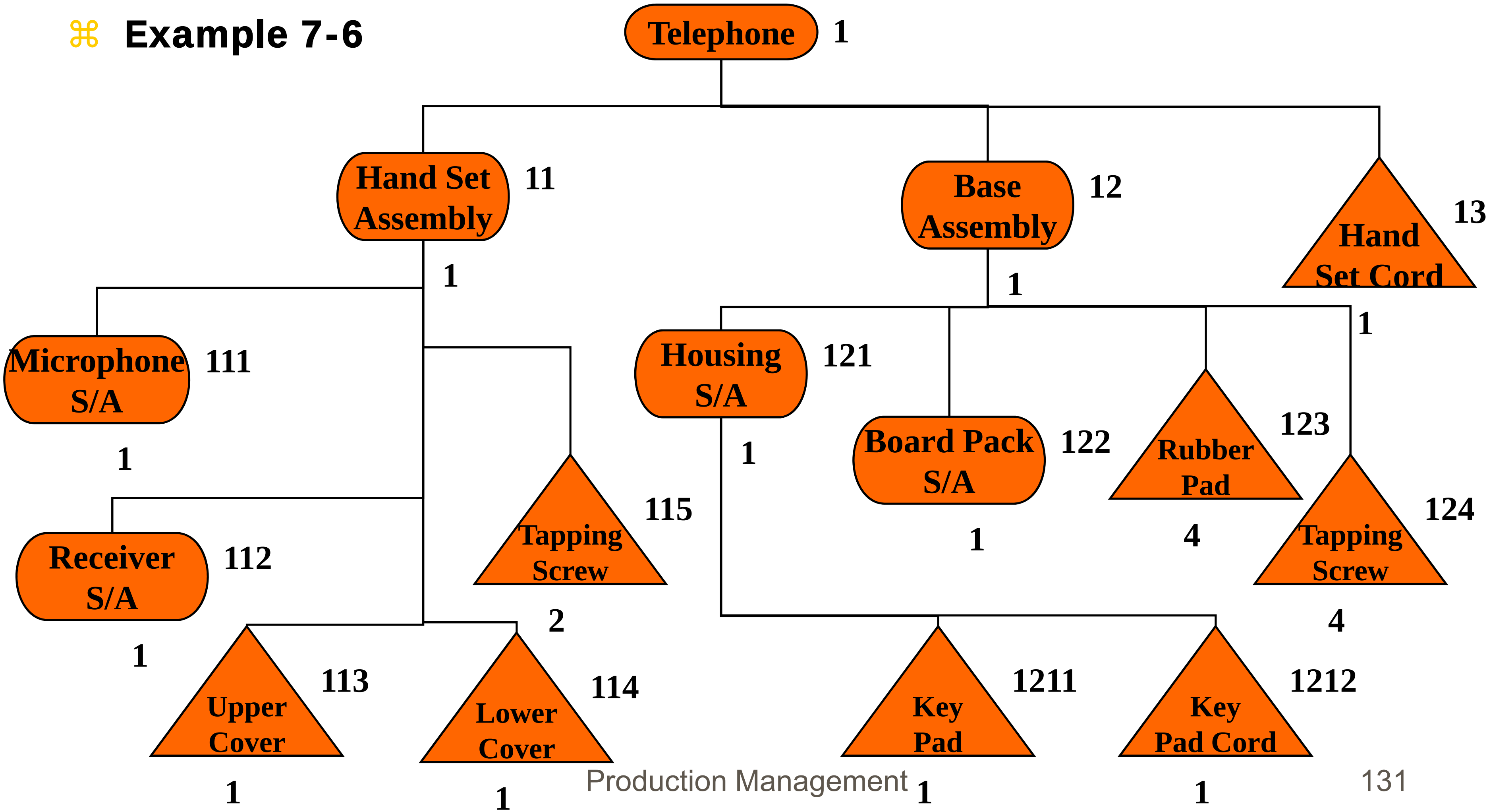


Material Requirements Planning

- **MRP Process**
  - goal is to find net requirements (trigger purchase and work orders)
  - explosion
    - Example:
    - MPS, 100 end items
    - yields gross requirements
  - netting
    - Net requirements = Gross requirements - on hand inventory - quantity on order
    - done at each level prior to further explosion
  - offsetting
    - the timing of order release is determined
  - lotsizing
    - batch size is determined

Material Requirements Planning

- **Example 7-6**



Material Requirements Planning

PART 11 (gross requirements given)
net requirements?
Planned order release?
Net requ.(week 2) = 600 – (1600 + 700) = -1700 =>Net requ.(week2) = **0**
Net requ.(week 3) = 1000 – (1700 + 200) = -900 =>Net requ.(week3) = **0**
Net requ.(week 4) = 1000 – 900 = **100** etc.

	week								
	current	1	2	3	4	5	6	7	8
gross requirements			600	1000	1000	2000	2000	2000	2000
scheduled receipts		400	700	200					
projected inventory balance	1200	1600	1700	900	0	0	0	0	0
net requirements					100	2000	2000	2000	2000
planned receipts									
planned order release									

Material Requirements Planning

Assumptions:
lot size: 3000
lead time: 2 weeks

	week								
	current	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
gross requirements			600	1000	1000	2000	2000	2000	2000
scheduled receipts		400	700	200					
projected inventory balance	1200	1600	1700	900	**2900**	**900**	**1900**	**2900**	**900**
net requirements					100	2000	2000	2000	2000
planned receipts					3000		3000	3000	
planned order release			3000		3000	3000			

Material Requirements Planning

- **Multilevel explosion**

part number	description	Qty
12	base assembly	1
121	housing S/A	1
123	rubber pad	4
1211	key pad	1

  - lead time is one week
  - lot for lot for parts 121, 123, 1211
  - part 12: fixed lot size of 3000

Part 12	current	1	2	3	4	5	6	7	8
gross requirements			600	1000	1000	2000	2000	2000	2000
scheduled receipts		400	400	400					
projected inventory balance	800	1200	1000	400	2400	400	1400	2400	400
net requirements		0	0	0	0	0	0	0	0
planned receipts		0	0	0	3000	0	3000	3000	0
planned order release	0	0	0	3000	0	3000	3000	0	0

Part 121	current	1	2	3	4	5	6	7	8
gross requirements	0	0	0	3000	0	3000	3000	0	0
scheduled receipts									
projected inventory balance	500	500	500	0	0	0	0	0	0
net requirements		0	0	0	0	0	0	0	0
planned receipts		0	0	2500	0	3000	3000	0	0
planned order release		0	2500	0	3000	3000	0	0	0

Part 123	current	1	2	3	4	5	6	7	8
gross requirements	0	0	0	12000	0	12000	12000	0	0
scheduled receipts			10000						
projected inventory balance	15000	15000	25000	13000	13000	1000	0	0	0
net requirements		0	0	0	0	0	0	0	0
planned receipts		0	0	0	0	0	11000	0	0
planned order release		0	0	0	0	11000	0	0	0

Part 1211	current	1	2	3	4	5	6	7	8
gross requirements	0	0	2500	0	3000	3000	0	0	0
scheduled receipts		1500							
projected inventory balance	1200	2700	200	200	0	0	0	0	0
net requirements		0	0	0	0	0	0	0	0
planned receipts		0	0	0	2800	3000	0	0	0
planned order release		0	0	2800	3000	0	0	0	0

Material Requirements Planning

- **MRP Updating Methods**
  - MRP systems operate in a dynamic environment
  - regeneration method: the entire plan is recalculated
  - net change method: recalculates requirements only for those items affected by change

	February				Updated MPS for February			
	Week				Week			
Product	5	6	7	8	5	6	7	8
A	2000	2000	2000	2000	2000	2000	2300	1900
B	350	-	-	350	500	-	200	150
C	1000	-	1000	1000	1000	-	800	1000
D	-	300	200	-	-	300	200	-

	Net Change for February			
	Week			
Product	5	6	7	8
A			300	-100
B	150		200	-200
C			-200	
D				

Material Requirements Planning

- **Additional Netting procedures**
  - implosion:
    - opposite of explosion
    - finds common item
  - combining requirements:
    - process of obtaining the gross requirements of a common item
  - pegging:
    - identify the item's end product
    - useful when item shortages occur

Material Requirements Planning

- **Lot Sizing in MRP**
  - minimize set-up and holding costs
  - can be formulated as MIP
  - a variety of heuristic approaches are available
  - simplest approach: use independent demand procedures (e.g. EOQ) at every level

Material Requirements Planning

- **MIP Formulation**
  - Indices:

$i = 1 \ldots P$	label of each item in BOM (assumed that all labels are sorted with respect to the production level starting from the end-items)
$t = 1 \ldots T$	period t
$m = 1 \ldots M$	resource m

  - Parameters:

$\Gamma(i)$	set of immediate successors of item i
$\Gamma^{-1}(i)$	set of immediate predeccessors of item i
s_i	setup cost for item i
c_{ij}	quantity of itme i required to produce item j
h_i	holding cost for one unit of item i
a_{mi}	capacity needed on resource m for one unit of item i
b_{mi}	capacity needed on resource m for the setup process of item i
L_{mt}	available capacity of resource m in period t
oc_m	overtime cost of resource m
G	large number, but as small as possible (e.g. sum of demands)
D_{it}	external demand of item i in period t

Material Requirements Planning

- Decision variables:

x_{it}	deliverd quantity of item *i* in period *t*
I_{it}	inventory level of item *i* at the end of period *t*
O_{mt}	overtime hours required for machine *m in* period *t*
y_{it}	binary variable indicating if item *i* is produced in period *t* (=1) or not (=0)

- Equations:

$$\min \sum_{i=1}^{P}\sum_{t=1}^{T}(s_i y_{it} + h_i I_{it}) + \sum_{t=1}^{T}\sum_{m=1}^{M} oc_m O_{mt}$$

$$I_{i,t} = I_{i,t-1} + x_{i,t} - \sum_{j\in\Gamma(i)} c_{ij} x_{jt} - D_{it} \quad \forall i,t \qquad x_{it} - G y_{it} \le 0 \quad \forall i,t$$

$$\sum_{i=1}^{P}(a_{mi} x_{it} + b_{mi} y_{it}) \le L_{mt} + O_{mt} \quad \forall m,t \qquad x_{it}, I_{it}, O_{mt} \ge 0, \quad y_{it} \in \{0,1\} \quad \forall i,m,t$$

Material Requirements Planning

- **Multi-Echelon Systems**
  - Multi-echelon inventory
  - each level is referred as an **echelon**
  - “total inventory in the system varies with the number of stocking points”
  - Modell (Freeland 1985):
    - demand is insensitive to the number of stocking points
    - demand is normally distributed and divided evenly among the stocking points,
    - demands at the stocking points are independent of one another
    - a (Q,R) inventory policy is used
    - β-Service level (fill rate) is applied
    - Q is determined from the EOQ formula

Material Requirements Planning

- **Reorder point in (Q,R) policies:**
  - i: total annual inventory costs (%)
  - c: unit costs
  - A: ordering costs
  - τ :lead time
  - σ_τ: variance of demand in lead time

- **given a fill rate β choose $z(\beta)$ such that:**

$$L(z) = \int_z^\infty (y - z)\,\phi(y)dy = \frac{(1-\beta)Q}{\sigma_\tau}$$

ϕ: density of ***N(0,1)*** distribution; $L(z)$: standard loss function

Unit Normal Linear Loss Integral L(Z)

Z	.00	.02	.04	.06	.08
0.00	.3989	.3890	.3793	.3697	.3602
0.10	.3509	.3418	.3329	.3240	.3154
0.20	.3069	.2986	.2904	.2824	.2745
0.30	.2668	.2592	.2518	.2445	.2374
0.40	.2304	.2236	.2170	.2104	.2040
0.50	.1978	.1917	.1857	.1799	.1742
0.60	.1687	.1633	.1580	.1528	.1478
0.70	.1429	.1381	.1335	.1289	.1245
0.80	.1202	.1160	.1120	.1080	.1042
0.90	.1004	.0968	.0933	.0899	.0866
1.00	.0833	.0802	.0772	.0742	.0714
1.10	.0686	.0660	.0634	.0609	.0585
1.20	.0561	.0539	.0517	.0496	.0475
1.30	.0456	.0437	.0418	.0401	.0383
1.40	.0367	.0351	.0336	.0321	.0307
1.50	.0293	.0280	.0268	.0256	.0244
1.60	.0233	.0222	.0212	.0202	.0192
1.70	.0183	.0174	.0166	.0158	.0150
1.80	.0143	0..0136	.0129	.0122	.0116
1.90	.0110	.0104	.0099	.0094	.0089
2.00	.0084	.0080	.0075	.0071	.0067
2.10	.0063	.0060	.0056	.0053	.0050
2.20	.0047	.0044	.0042	.0039	.0037
2.30	.0036	.0034	.0032	.0030	.0028
2.40	.0027	.0026	.0024	.0023	.0022
2.50	.0021	.0018	.0017	.0016	.0016

Material Requirements Planning

- **Safety stock:** $s = z \cdot \sigma_\tau$
- **Reorder point:** $R = \overline{D}_\tau + z \cdot \sigma_\tau$
- **Order quantity:** $Q = EOQ = \sqrt{\frac{2A\overline{D}}{ic}}$
- **Average inventory:** $\bar{I}(1) = \frac{Q}{2} + s$

$\bar{I}(n) =$ average inventory for n stocking points

$$\bar{I}(1) = \frac{1}{2}\sqrt{\frac{2A\overline{D}}{ic}} + z\sigma_\tau$$

Material Requirements Planning

for two stocking points:

demand at each point: D/2

variance of lead-time demand: $\sigma_\tau^2 / 2$

standard deviation is: $\sigma_\tau / \sqrt{2}$

average inventory at each stocking point is:

$$\frac{1}{2}\sqrt{\frac{2A\overline{D}/2}{ic}} + \frac{z\sigma_\tau}{\sqrt{2}} = \frac{1}{\sqrt{2}}(Q/2 + s)$$

Material Requirements Planning

the average inventory for two stocking point is :

$$\bar{I}(2) = 2\left[\frac{1}{\sqrt{2}}(Q/2+s)\right] = \sqrt{2}\left(Q/2+s\right) = \sqrt{2}\cdot\bar{I}(1)$$

$$\bar{I}(n) = \sqrt{n}\cdot\bar{I}(1)$$

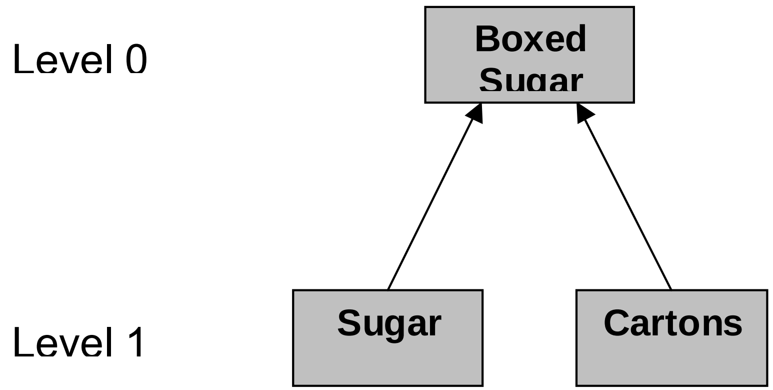
for each level the safety stock is : s/$\sqrt{n}$

the total safety stock is $\sqrt{n}\cdot s$

Material Requirements Planning

Example: At the packaging department of a sugar refinery:

A very-high-grade powdered sugar:



Sugar-refining lead time is five days;
Production lead time (filling time) is negligible;
Annual demand: D = 800 tons and σ= 2,5
Lead-time demand is normally distributed with $D\tau$ = 16 tons and $\sigma\tau$ = 3,54 tons
Fill rate = 95%
A = $50, c = $4000, i = 20%

Material Requirements Planning

Inventory at level 0 and 1? Safety stock?

$$Q = \sqrt{\frac{2AD}{ic}} = \sqrt{\frac{2x50x800}{800}} = 10\ tons$$

ß = 0,95 => z = 0,71

s = zστ = 0,71x3,54 = 2,51 tons

Suppose we keep inventory in level 0 only, i.e., n = 1:

$$I(1) = \frac{Q}{2} + s = \frac{10}{2} + 2,51 = 7,51 tons$$

Suppose inventory is maintained at both level 0 and level 1, i.e., n = 2:

$$I(2) = \sqrt{2I(1)} = 10,62\ tons$$

The safety stock in each level is going to be:

$$\frac{s}{\sqrt{2}} = \frac{2,51}{\sqrt{2}} = 1,77 tons$$

Material Requirements Planning

- **MRP as Multi-Echelon Inventory Control**
  - continuous-review type policy (Q,R)
  - hierarchy of stocking points (installation)
  - installation stock policy
  - echelon stock (policy): installation inventory position plus all downstream stock
  - MRP:
    - rolling horizon
    - level by level approach
    - bases ordering decisions on projected future installation inventory level

Material Requirements Planning

- All demands and orders occur at the beginning of the time period
- orders are initiated immediately after the demands, first for the final items and then successively for the components
- all demands and orders are for an integer number of units
- T= planning horizon
- τ_i= lead time for item i
- s_i= safety stock for item I
- R_i= reorder point for item I
- Q_i=Fixed order quantity of item i
- D_{it}= external requirements of item i in period t

Material Requirements Planning

- **Installation stock policies (Q,R^i) for MRP:**
  - a production order is triggered if the installation stock minus safety stock is insufficient to cover the requirements over the next τ_i periods
  - an order may consist of more than one order quantity Q

  - if lead time τ_i = 0, the MRP is equal to an installation stock policy.
  - safety stock = reorder point

Material Requirements Planning

- **Echelon stock policies (Q,R^e) for MRP:**
  - Consider a serial assembly system
  - Installation 1 is the downstream installation (final product)
  - the output of installation i is the input when producing one unit of item i-1 at the immediate downstream installation
  - w_i = installation inventory position at installation i
  - I_i = echelon inventory position at installation i (at the same moment)

- $$I_i = w_i + w_{i-1} + \dots w_1$$

  - a multi-echelon (Q,R) policy is denoted by (Q_i, R_i^e)
  - R_i^e gives the reorder point for echelon inventory at i

Material Requirements Planning

$$R_1^e = s_1 + D\tau_1$$
$$R_i^e = s_i + D\tau_i + R_{i-1}^e + Q_{i-1}$$

- **Example:**
- **Two-level system, 6 periods**

  $I_1^0 = 18,\ I_2^0 = 38,\ R_1^e = 20,\ R_2^e = 34,\ Q_1 = 10,\ Q_2 = 30$
- **D = 2 (Item 1), $\tau_1 = 1$, $\tau_2 = 2$**
-

Material Requirements Planning

	Period	1	2	3	4	5	6
	Demand	2	2	2	2	2	2
Item 1	Level w1	18	26	24	22	20	28
	Production	10	0	0	0	10	0
Item 2	Level w2	10	10	10	10	30	10
	Production	0	0	30	0	0	0

Suppose now that five units were demanded in period 2:

	Period	2	3	4	5	6	7
	Demand	5	2	2	2	2	2
Item 1	Level w1	23	21	19	27	25	23
	Production	0	0	10	0	10	0
Item 2	Level w2	10	10	30	30	30	30
	Production	30	0	0	0	0	0

Material Requirements Planning

- **Lot Size and Lead Time**
  - lead time is affected by capacity constraints
  - lot size affects lead time
- **batching effect**
  - an increase in lot size should increase lead time
- **saturation effect**
  - when lot size decreases, and set-up is not reduced, lead time will increase

- **expected lead time can be calculated using models from queueing theory (M/G/1)**

Material Requirements Planning

$$L = \text{lead time}$$

$$L = \frac{(\lambda / \mu)^2 + \lambda^2\sigma^2}{2\lambda(1-\lambda/\mu)} + \frac{1}{\mu}$$

$$\lambda = \text{mean arrival rate}$$

$$\mu = \text{mean service rate}$$

$$\sigma^2 = \text{service time variance}$$

Material Requirements Planning

D_j = demand per period for product j

t_j = unit-production time for product j

S_j = set-up time for product j

Q_j = lotsize for product j

mean arrival rate of batches:

$$\lambda = \sum_{j=1}^{n} \lambda_j = \sum_{j=1}^{n} \frac{D_j}{Q_j}$$

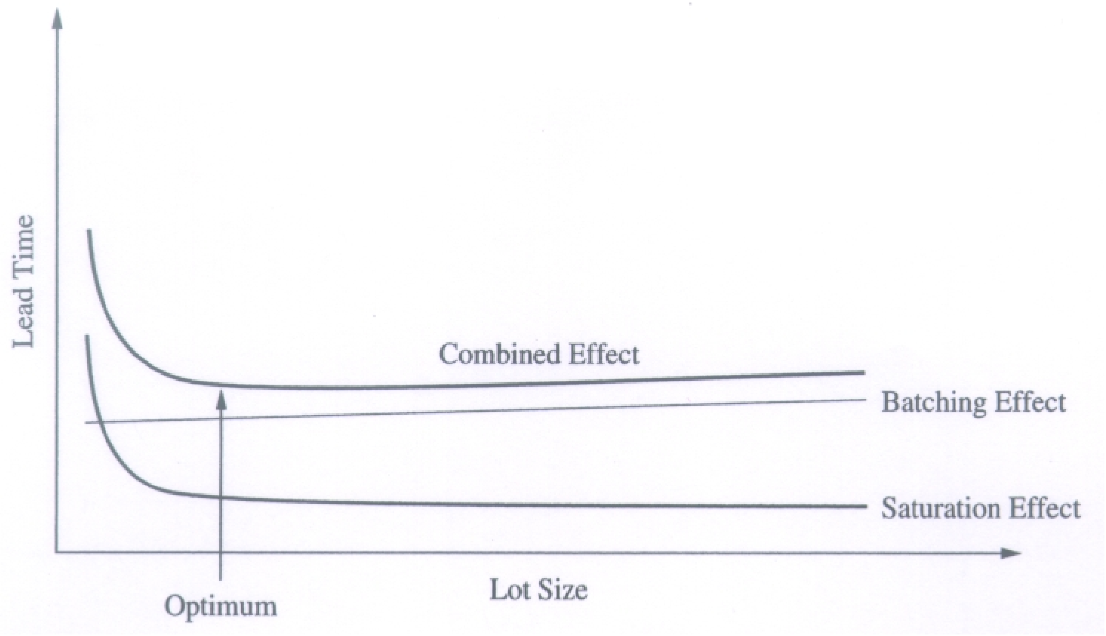
mean service time:

$$\frac{1}{\mu} = \frac{\sum_{j=1}^{n} \lambda_j (S_j + t_j Q_j)}{\sum_{j=1}^{n} \lambda_j}$$

service-time variance:

$$\sigma^2 = \frac{\sum_{j=1}^{n} \lambda_j (S_j + t_j Q_j)^2}{\sum_{j=1}^{n} \lambda_j} - \left(\frac{1}{\mu}\right)^2$$

Material Requirements Planning



Material Planning

- **Work to do: 7.7ab, 7.8, 7.10, 7.11, 7.14 (additional information: available hours: 225 (Paint), 130 (Mast), 100 (Rope)), 7.15, 7.16, 7.17, 7.31-7.34**