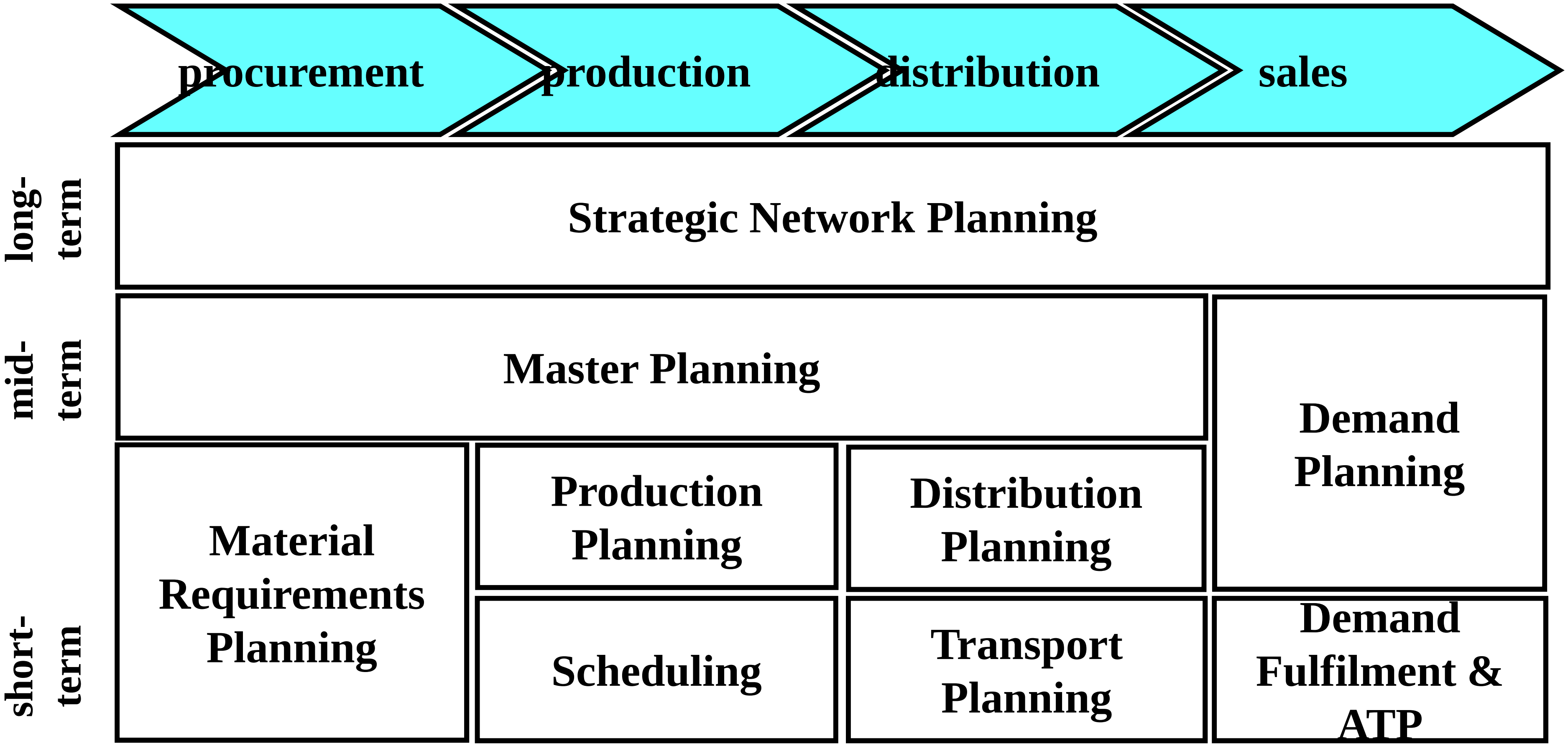
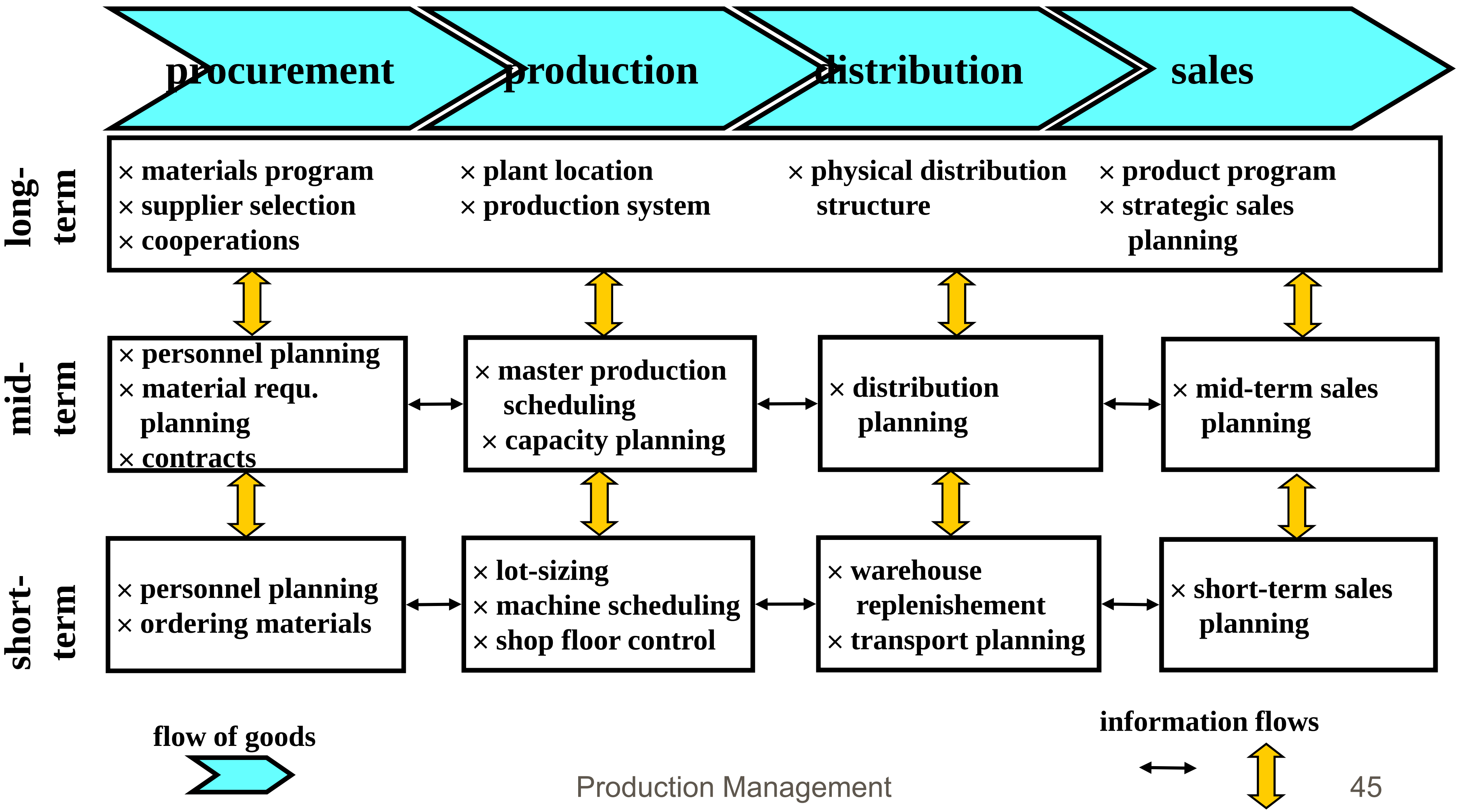
Chapter 5

Aggregate Planning

Supply Chain Planning Matrix



Supply Chain Planning Matrix



Aggregate Planning

- **Example:**
  - one product (plastic case)
  - two injection molding machines, 550 parts/hour
  - one worker, 55 parts/hour
  - steady sales 80.000 cases/month
  - 4 weeks/month, 5 days/week, 8h/day
  - how many workers?

- **in real life constant demand is rare**
  - change demand
  - produce a constant rate anyway
  - vary production

Aggregate Planning

- **Influencing demand**
  - do not satisfy demand
  - shift demand from peak periods to nonpeak periods
  - produce several products with peak demand in different period

- **Planning Production**
  - Production plan: how much and when to make each product
  - rolling planning horizon
  - long range plan
  - intermediate-range plan
    - units of measurements are aggregates
    - product family
    - plant department
    - changes in workforce, additional machines, subcontracting, overtime,…
  - Short-term plan

Aggregate Planning

- **Aspects of Aggregate Planning**
  - Capacity: how much a production system can make
  - Aggregate Units: products, workers,…
  - Costs
    - production costs (economic costs!)
    - inventory costs(holding and shortage)
    - capacity change costs

Aggregate Planning

- **Spreadsheet Methods**
- **Zero Inventory Plan**
  - Precision Transfer, Inc. Produces more than 300 different precision gears (the aggregation unit is a gear!).
  - Last year (=260 working days) Precision made 41.383 gears of various kinds with an average of 40 workers.
  - 41.383 gears per year
  - *40 x 260 worker-days/year = 3,98 -> 4 gears/ worker-day*

  - Aggregate demand forecast for precision gear:

Month	January	February	March	April	May	June	Total
Demand	2760	3320	3970	3540	3180	2900	19.670

Aggregate Planning

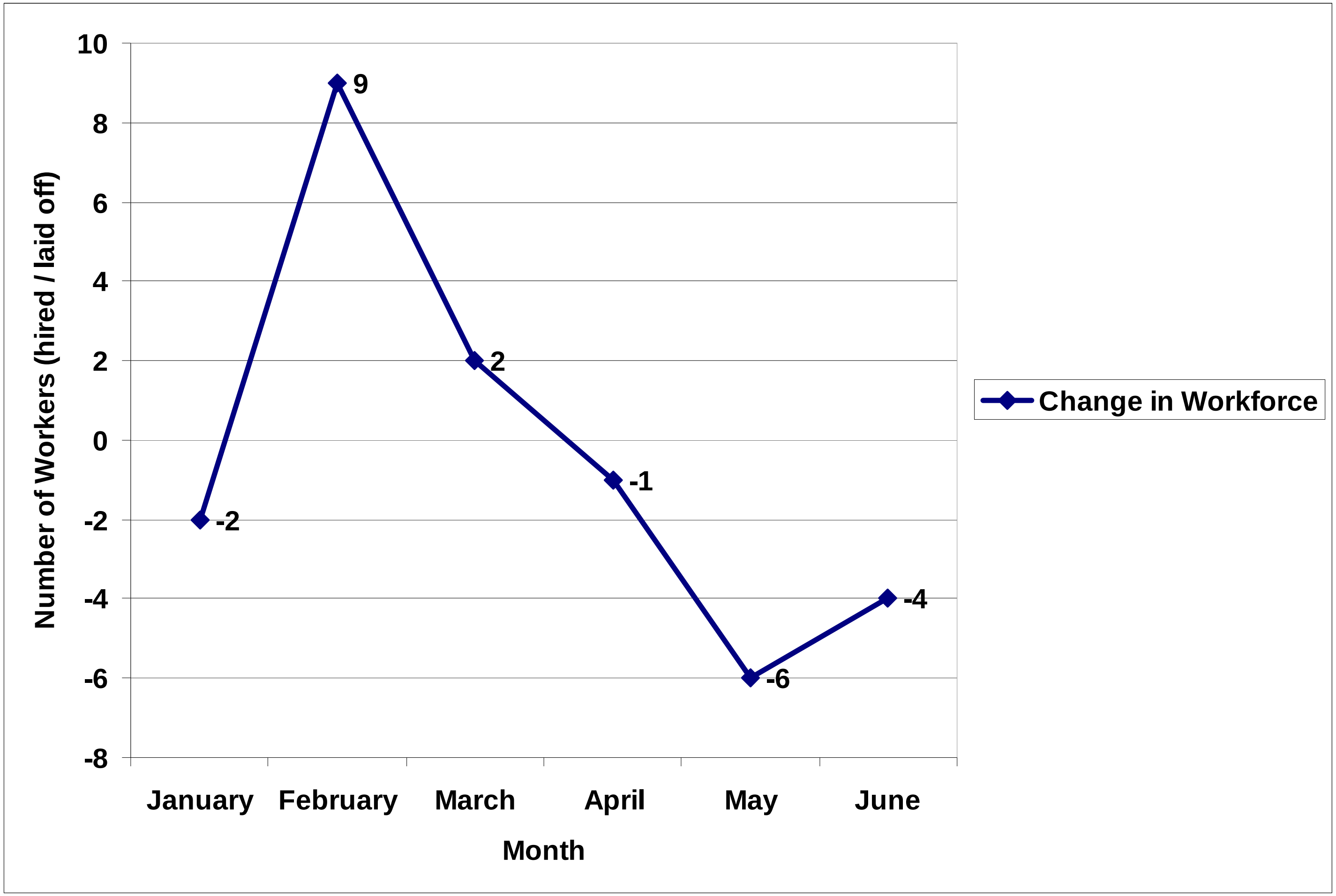
- holding costs: $5 per gear per month
- backlog costs: $15 per gear per month
- hiring costs: $450 per worker
- lay-off costs: $600 per worker
- wages: $15 per hour (all workers are paid for 8 hours per day)
- there are currently 35 workers at Precision
- currently no inventory

- Production plan?

Aggregate Planning

- **Zero Inventory Plan**
  - produce exactly amount needed per period
  - adapt workforce

Aggregate Planning



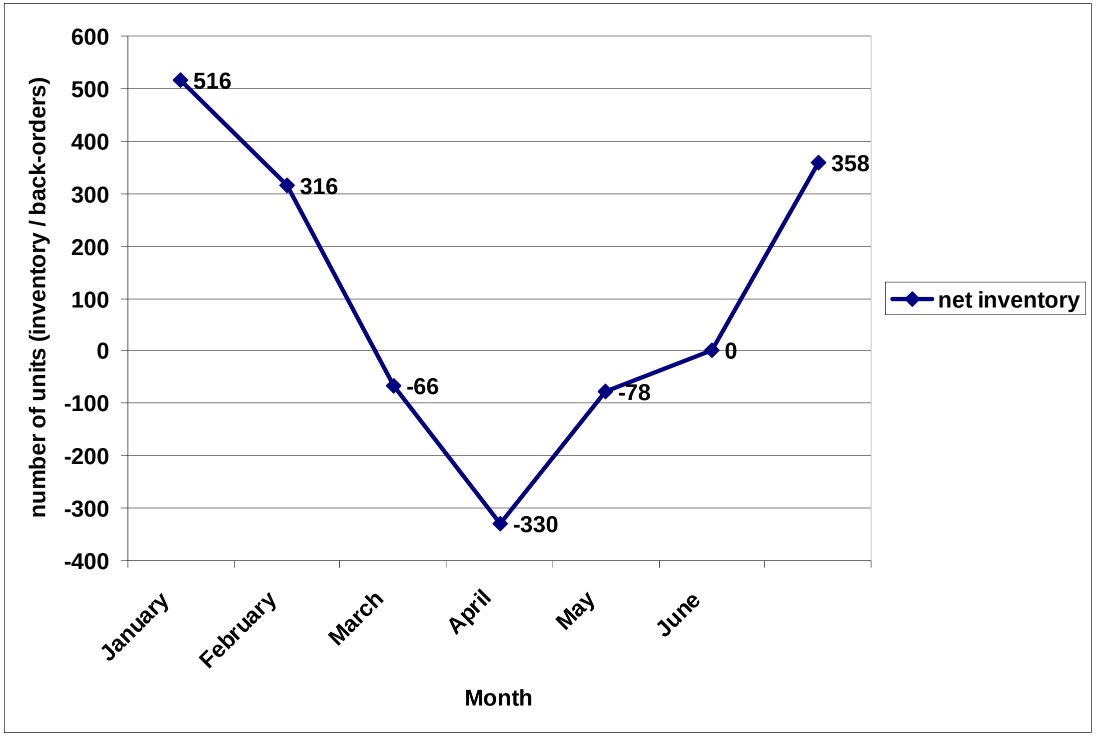
Aggregate Planning

- **Level Work Force Plan**
  - backorders allowed
  - constant numbers of workers
  - demand over the planning horizon
  - gears a worker can produce over the horizon

  - 19670/(4x129)=38,12 -> 39 workers are always needed

Aggregate Planning

- Inventory: January: 3276 - 2760 = 516
- February: 516 + 3120 – 3320
- March: 316 + 3588 – 3670 = -66! -Backorders: 66 x $15 = $990



Aggregate Planning

- **no backorders are allowed**
  - *workers= cumulative demand/(cumulative days x units/workers/day)*

  - January: 2760/(21 x 4) = 32,86 -> 33 workers
  - February: (2760+3320)/[(21+20) x 4] = 37,07 -> 38 workers.
  - March: 10.050/(64 x 4) =>40 workers
  - April: 13.590/(85 x 4) => 40 workers
  - May: 16.770/(107 x 4) => 40 workers
  - June: 19670/(129 x 4) => 39 workers

Aggregate Planning

- **Example Mixed Plan**
  - The number of workers used is an educated guess based on the zero inventory and level work force plans!

Spreadsheet Methods Summary

	Zero-Inv.	Level/BO	Level/No BO	Mixed
Hiring cost	4950	1800	2250	3150
Lay-off cost	7800	0	0	4200
Labor cost	59856	603720	619200	593520
Holding cost	0	4160	6350	3890
BO cost	0	7110	0	990
Total cost	611310	616790	627800	605180
Workers	33	39	40	35

Aggregate Planning

- **Linear Programming Approaches to Aggregate Planning**

  Parameters:

  T... Planning horizon length

  t ... Index of periods, t=1,2,..., T

  D_t …forcasted number of units demanded in period t

  n_t …number of units that can be made by one worker in period t

  C_t^P …cost to produce one unit in period t

  C_t^W …cost of one worker in period t

Aggregate Planning

C_t^H …cost to hire one worker in period t

C_t^L …cost to lay off one worker in period t

C_t^I …cost to hold one unit in inventory in period t

C_t^B …cost to backorder one unit in period t

Aggregate Planning

Decision Variables:

$P_t \ldots$ number of units produced in period t

$W_t \ldots$ number of workers available in period t

$H_t \ldots$ number of workers hired in period t

$L_t \ldots$ number of workers laid off in period t

$I_t \ldots$ number of units held in inventory in period t

$B_t \ldots$ number of units backordered in period t

Aggregate Planning

Constraints: work, Capacity, force, material

$$\mathrm{P_t} \le n_t W_t \qquad t = 1, 2, \ldots, T$$

$$\mathrm{W_t} = W_{t-1} + H_t - L_t \qquad t = 1, 2, \ldots, T$$

net inventory this period = net inventory last period + production this period - demand this period

$$\mathrm{I_t} - B_t = \mathrm{I_{t\text{-}1}} - B_{t-1} + P_t - D_t$$

Costs

$$\sum_{t=1}^{T} (C_t^P P_t + C_t^W W_t + C_t^H H_t + C_t^L L_t + C_t^I I_t + C_t^B B_t)$$

Aggregate Planning

- **Example: Precision Transfer**
  - Planning horizon: 6 months T= 6
  - Costs do not vary over time $C_t^P = 0$
  - d_t : days in month t
  - $C_t^W = \$120d_t$
  - $C_t^H = \$450$
  - $C_t^L = \$600$
  - $C_t^I = \$5$

  - We assume that no backorders are allowed!
  - no production costs and no backorder costs are included!
  - Demand

January	February	March	April	May	June	Total
2760	3320	3970	3540	3180	2900	19.670

Linear Program Model for Precision Transfer

Minimize

$$
\begin{aligned}
&2520W_1 + 2400W_2 + 2760W_3 + 2520W_4 + 2640W_5 + 2640W_6 \\
&+ 450(H_1 + H_2 + H_3 + H_4 + H_5 + H_6) \\
&+ 600(L_1 + L_2 + L_3 + L_4 + L_5 + L_6) \\
&+ 5(I_1 + I_2 + I_3 + I_4 + I_5 + I_6)
\end{aligned}
$$

subject to
(Production-capacity constraints)

$$P_1 \le 84W_1, \quad P_2 \le 80W_2, \quad P_3 \le 92W_3, \quad P_4 \le 84W_4, \quad P_5 \le 88W_5, \quad P_6 \le 88W_6,$$

(Work-force constraints)

$$
\begin{aligned}
&W_1 = 35 + H_1 - L_1, \quad W_2 = W_1 + H_2 - L_2, \quad W_3 = W_2 + H_3 - L_3, \\
&W_4 = W_3 + H_4 - L_4, \quad W_5 = W_4 + H_5 - L_5, \quad W_6 = W_5 + H_6 - L_6,
\end{aligned}
$$

(Inventory-balance constraints)

$$
\begin{aligned}
&I_1 = P_1 - 2760, \quad I_2 = I_1 + P_2 - 3320, \quad I_3 = I_2 + P_3 - 3970, \\
&I_4 = I_3 + P_4 - 3540, \quad I_5 = I_4 + P_5 - 3180, \quad I_6 = I_5 + P_6 - 2900\,(= 0),
\end{aligned}
$$

(Non-negativity constraints)

$$
\begin{aligned}
&P_1, P_2, P_3, P_4, P_5, P_6, W_1, W_2, W_3, W_4, W_5, W_6, H_1, H_2, H_3, H_4, H_5, H_6, \\
&L_1, L_2, L_3, L_4, L_5, L_6, I_1, I_2, I_3, I_4, I_5, I_6 \ge 0
\end{aligned}
$$

Aggregate Planning

- **LP solution (total cost = $600 191,60)**

	Production	Inventory	Hired	Laid off	Workers
January	*2940,00*	*180,00*	*0,00*	*0,00*	*35,00*
February	*3232,86*	*92,86*	*5,41*	*0,00*	*40,41*
March	*3877,14*	*0,00*	*1,73*	*0,00*	*42,14*
April	*3540,00*	*0,00*	*0,00*	*0,00*	*42,14*
May	*3180,00*	*0,00*	*0,00*	*6,01*	*36,14*
June	*2900,00*	*0,00*	*0,00*	*3,18*	*32,95*

Aggregate Planning

- **Rounding LP solution**

	January	February	March	April	May	June	Total
Days	21	20	23	21	22	22	129
Units/Worker	84	80	92	84	88	88	516
Demand	2760	3320	3970	3540	3180	2900	19670
Workers	**35**	**41**	**42**	**42**	**36**	**33**	229
Capacity	2940	3280	3864	3528	3168	2904	19684
Capacity - Demand	180	-40	-106	-12	-12	4	14
Cumulative Difference	180	140	34	22	10	14	400
Produced	2930	3280	3864	3528	3168	2900	19670
Net inventory	170	130	24	12	0	0	336
Hired	0	6	1	0	0	0	7
Laid Off	0	0	0	0	6	3	9
Costs	89050	101750	116490	105900	98640	88920	**600750**

Aggregate Planning

- **Practical Issues**
  - 100.000 variables and 40.000 constraints
  - LP/MIP Solvers: CPLEX, XPRESS-MP, ...
- **Extensions**
  - Bounds

$$\mathrm{I_t} \le I_t^U$$

$$\mathrm{I_t^L} \le \mathrm{I_t} \le I_t^U$$

$$\mathrm{L_t} \le 0.05 W_t$$

Training

$$W_t = W_{t-1} + H_{t-1} - L_t$$

Aggregate Planning

- **Transportation Models**
  - supply points: periods, initial inventory
  - demand points: periods, excess demand, final inventory

$$n_t W_t = \text{capacity during period t}$$

$$D_t = \text{forecasted number of units demanded in period t}$$

$$\mathrm{C}_{\mathrm{t}}^{\mathrm{P}} = \text{the cost to produce one unit in period t}$$

$$\mathrm{C}_{\mathrm{t}}^{\mathrm{I}} = \text{the cost to hold one unit in inventory in period t}$$

Aggregate Planning

t	1	2	3
capacity $n_t W_t$	350	300	350
demand	200	300	400
production costs	10	11	12
holding costs	2	2	2

initial inventory: 50
final inventory: 75

Aggregate Planning

	1	2	3	Ending inventory	Excess capacity	Available capacity
Beginning inventory	0 **50**	2	4	6	0	50
Period 1	10 **150**	12	14 **50**	16 **75**	0 **75**	350
Period 2	-	11 **300**	13	15	0	300
Period 3	-	-	12 **350**	14	0	350
Demand	200	300	400	75	75	1050

Aggregate Planning

- Extension:

t	1	2	3
capacity $n_t W_t$	350	350	300
demand	400	300	400
production costs	10	11	12
holding costs	2	2	2

  - **overtime**: overtime capacity is **90, 90** and **75** in period 1, 2 and 3;
  - overtime costs are **$16, $18** and **$ 20** for the three periods respectively;
  - **backorders**:units can be backordered at a cost of **$5** per unit-month; production in period 2 can be used to satisfy demand in period 1

Aggregate Planning

		1	**2**	**3**	Ending inventory	Excess capacity	Available capacity
Beginning inventory		0	2 **25**	4 **25**	6	0	50
Period 1	Regular time	10 **350**	12	14	16	0	350
	Overtime	16 **50**	18	20	22	0 **40**	90
Period 2	Regular time	16	11 **275**	13	15 **75**	0	350
	Overtime	23	18	20	22	0 **90**	90
Period 3	Regular time	22	17	12 **300**	14	0	300
	Overtime	30	25	20 **75**	22	0	75
Demand		400	300	400	75	130	1305

Aggregate Planning

- **Disaggregating Plans**
  - aggregate units are not actually produced, so the plan should consider individual products
  - disaggregation
  - master production schedule

- **Questions:**
  - In which order should individual products be produced?
    - e.g.: shortest run-out time $R_i = I_i / D_i$
  - How much of each product should be produced?
    - e.g.: balance run-out time

Aggregate Planning

- **Advanced Production Planning Models**
  - Multiple Products
  - same notation as before
  - add subscript i for product i

  - Objective function

$$\min \sum_{t=1}^{T} \left(C_t^W W_t + C_t^H H_t + C_t^L L_t + \sum_{i=1}^{N} C_{it}^P P_{it} + C_{it}^I I_{it} \right)$$

Aggregate Planning

subject to

$$\sum_{i=1}^{N} \left(\frac{1}{n_{it}} \right) P_{it} \leq W_t \qquad t = 1, 2, \ldots, T$$

$$W_t = W_{t-1} + H_t - L_t \qquad \text{t=1,2,...,T}$$

$$I_{it} = I_{it-1} + P_{it} - D_{it} \qquad \text{t=1,2,...,T; i=1,2,...,N}$$

$$P_{it}, W_t, H_t, L_t, I_{it} \geq 0 \qquad \text{t=1,2,...,T; i=1,2,...,N}$$

Aggregate Planning

- **Computational Effort:**
  - 10 products, 12 periods: 276 variables, 144 constraints
  - 100 products, 12 periods: 2436 variables, 1224 constaints

Aggregate Planning

- Example: Carolina Hardwood Product Mix
  - Carolina Hardwood produces 3 types of dining tables;
  - There are currently 50 workers employed who can be hired and laid off at any time;
  - Initial inventory is 100 units for table1, 120 units for table 2 and 80 units for table 3;

t	1	2	3	4
costs of hiring	420	410	420	405
costs of lay off	800	790	790	800
costs per worker	600	620	620	610

Aggregate Planning

- The number of units that can be made by one worker per period:

t	Table 1	Table 2	Table 3
1	200	300	260
2	220	310	255
3	210	300	250
4	200	290	265

- Forecasted demand, unit cost and holding cost per unit are:

	Demand			Unit costs			Holding costs		
t	Table 1	Table 2	Table 3	Table 1	Table 2	Table 3	Table 1	Table 2	Table 3
1	3500	5400	4500	120	150	200	10	12	12
2	3100	5000	4200	125	150	210	9	11	12
3	3000	5100	4100	120	145	205	10	12	11

Aggregate Planning

Minimize

$$600W_1 + 620W_2 + 620W_3 + 610W_4 + 420H_1 + 410H_2 + 420H_3 + 405H_4 + 800L_1 + 790L_2 + 790L_3 + 800L_4$$
$$+ 120P_{11} + 150P_{21} + 200P_{31} + 125P_{12} + 150P_{22} + 210P_{32} + 120P_{13} + 145P_{23} + 205P_{33} + 125P_{14} + 148P_{24} + 205P_{34}$$
$$+ 10I_{11} + 12I_{21} + 12I_{31} + 9I_{12} + 11I_{22} + 12I_{32} + 10I_{13} + 12I_{23} + 11I_{33} + 10I_{14} + 11I_{24} + 11I_{34}$$

subject to

$$\frac{P_{11}}{200} + \frac{P_{21}}{300} + \frac{P_{31}}{260} \le W_1, \quad \frac{P_{12}}{220} + \frac{P_{22}}{310} + \frac{P_{32}}{255} \le W_2,$$
$$\frac{P_{13}}{210} + \frac{P_{23}}{300} + \frac{P_{33}}{250} \le W_3, \quad \frac{P_{14}}{200} + \frac{P_{24}}{290} + \frac{P_{34}}{265} \le W_4,$$

$$W_1 = 50 + H_1 - L_1, \quad W_2 = W_1 + H_2 - L_2, \quad W_3 = W_2 + H_3 - L_3, \quad W_4 = W_3 + H_4 - L_4,$$

$$I_{11} = 100 + P_{11} - 3500, \quad I_{21} = 120 + P_{21} - 5400, \quad I_{31} = 80 + P_{31} - 4500,$$
$$I_{12} = I_{11} + P_{12} - 3100, \quad I_{22} = I_{21} + P_{22} - 5000, \quad I_{32} = I_{31} + P_{32} - 4200,$$
$$I_{13} = I_{12} + P_{13} - 3000, \quad I_{23} = I_{22} + P_{23} - 5100, \quad I_{33} = I_{32} + P_{33} - 4100,$$
$$I_{14} = I_{13} + P_{14} - 3400, \quad I_{24} = I_{23} + P_{24} - 5500, \quad I_{34} = I_{33} + P_{34} - 4600,$$

$$P_{it}, I_{it}, W_t, H_t, L_t, I_{it} \ge 0$$

Aggregate Planning

⌘ Multiple Products and Processes

T = horizon length, in periods
N = number of products
K = number of resource types
t = index of periods, $t = 1, 2, \ldots, T$
i = index of products, $i = 1, 2, \ldots, N$
k = index of resource types, $k = 1, 2, \ldots, K$
D_{it} = forecasted number of units demanded for product i in period t
m_i = number of different processes available to make product i
A_{kt} = amount of resource k available in period t
a_{ijk} = amount of resource k required by one unit of product i if produced by process j
C^P_{ijt} = cost to produce one unit of product i using process j in period t
C^I_{it} = cost to hold one unit of product i in inventory for period t

The decision variables are

P_{ijt} = number of units of product i produced by process j in period t
I_{it} = number of units of product i held in inventory at the end of period t

Aggregate Planning

The linear programming formulation is

$$\text{Minimize} \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{m_i} (C_{ijt}^{P} P_{ijt} + C_{it}^{I} I_{it})$$

subject to

$$\sum_{i=1}^{N} \sum_{j=1}^{m_i} a_{ijk} P_{ijt} \le A_{kt} \qquad t = 1, 2, \ldots, T; \;\; k = 1, 2, \ldots, K$$

$$I_{it} = I_{it-1} + \sum_{j=1}^{m_i} P_{ijt} - D_{it} \qquad t = 1, 2, \ldots, T; \;\; i = 1, 2, \ldots, N$$

$$P_{ijt}, I_{it} \ge 0 \qquad t = 1, 2, \ldots, T; \;\; i = 1, 2, \ldots, N; \;\; j = 1, 2, \ldots, m_i$$

Aggregate Planning

- Example: Cactus Cycles process plan
- CC produces 2 types of bicycles, street and road;
- Estimated demand and current inventory:

t	initial inventory	1	2	3
street b.	**100**	**1000**	**1050**	**1100**
road b.	**50**	**500**	**600**	**550**

- available capacity(hours) and holding costs per bike:

	Capacity(hours)		Holding	
t	Machine	Worker	Street	Road
1	8600	17000	5	6
2	8500	16600	6	7
3	8800	17200	5	7

Aggregate Planning

- process costs (process1, process2) and resource requirement per unit:

	Process1		Process2	
t	Street	Road	Street	Road
1	72	85	80	90
2	74	88	78	95
3	75	84	78	92
Machine hours required	5	8	4	6
Worker hours required	10	12	8	9

Aggregate Planning

Minimize

$$\begin{aligned}
&72P_{111} + 80P_{121} + 85P_{211} + 90P_{221} \\
&+ 74P_{112} + 78P_{122} + 88P_{212} + 95P_{222} \\
&+ 75P_{113} + 78P_{123} + 84P_{213} + 92P_{223} \\
&+ 5I_{11} + 6I_{12} + 5I_{13} + 6I_{21} + 7I_{22} + 7I_{23}
\end{aligned}$$

subject to

$$\begin{aligned}
&5P_{111} + 4P_{121} + 8P_{211} + 6P_{221} \le 8600, \quad && 10P_{111} + 8P_{121} + 12P_{211} + 9P_{221} \le 17000, \\
&5P_{112} + 4P_{122} + 8P_{212} + 6P_{222} \le 8500, \quad && 10P_{112} + 8P_{122} + 12P_{212} + 9P_{222} \le 16600, \\
&5P_{113} + 4P_{123} + 8P_{213} + 6P_{223} \le 8800, \quad && 10P_{113} + 8P_{123} + 12P_{213} + 9P_{223} \le 17200,
\end{aligned}$$

$$\begin{aligned}
&I_{11} = 100 + P_{111} + P_{121} - 1000, \quad && I_{21} = 50 + P_{211} + P_{221} - 500, \\
&I_{12} = I_{11} + P_{112} + P_{122} - 1050, \quad && I_{22} = I_{21} + P_{212} + P_{222} - 600, \\
&I_{13} = I_{12} + P_{113} + P_{123} - 1100, \quad && I_{23} = I_{22} + P_{213} + P_{223} - 550,
\end{aligned}$$

$$P_{ijt}, I_{it} \ge 0 \qquad t = 1, 2, 3; \quad i = 1, 2; \quad j = 1, 2$$

Aggregate Planning

- **solution: Objective Function value = $368,756.25**

	Street Bicycle			Road Bicycle		
	Process			Process		
t	1	2	Inventory	1	2	Inventory
1	900	0	0	118,75	525	193,75
2	1050	0	0	406,25	0	0
3	0	1100	0	550	0	0

Aggregate Planning - Extensions

- **Hopp/Spearman, S. 522-540**
- **Notation:**

$X_{it} \ldots$ amount of product i produced in period t

$r_i \ldots$ net profit from one unit of product i

$S_{it} \ldots$ amount of product i sold in period t

$a_{ij} \ldots$ time required on workstation j to produce one unit of product i

$c_{jt} \ldots$ capacity of workstation j in period t in units (consistent with a_{ij})

Aggregate Planning - Extensions

- **Backorders**

$$\max \sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} \mathrm{r_i} S_{it} - h_i I_{it}^{+} - \pi_i I_t^{-}$$

subject to

$$d_{it} \le S_{it} \le \bar{d}_{it} \qquad \text{for all } i,t$$

$$\sum_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{a_{ij}} X_{it} \le c_{jt} \qquad \text{for all } j,t$$

$$I_{it} = I_{it-1} + X_{it} - S_{it} \qquad \text{for all } i,t$$

$$I_{it} = I_{it}^{+} - I_{it}^{-} \qquad \text{for all } i,t$$

$$X_{it,} S_{it,} I_{it}^{+}, I_{it}^{-} \ge 0 \qquad \text{for all } i,t$$

Aggregate Planning - Extensions

- **Overtime**

$$l'_j = \text{cost of one hour of overtime at workstation j}$$

$$\mathrm{O}_{\mathrm{jt}} = \text{overtime at workstation j in period t in hours}$$

$$\max \sum_{\mathrm{t}=1}^{\bar{\mathrm{t}}} \left\{ \sum_{\mathrm{i}=1}^{\mathrm{m}} (\mathrm{r}_{\mathrm{i}} S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-}) - \sum_{j=1}^{n} l' \mathrm{O}_{\mathrm{jt}} \right\}$$

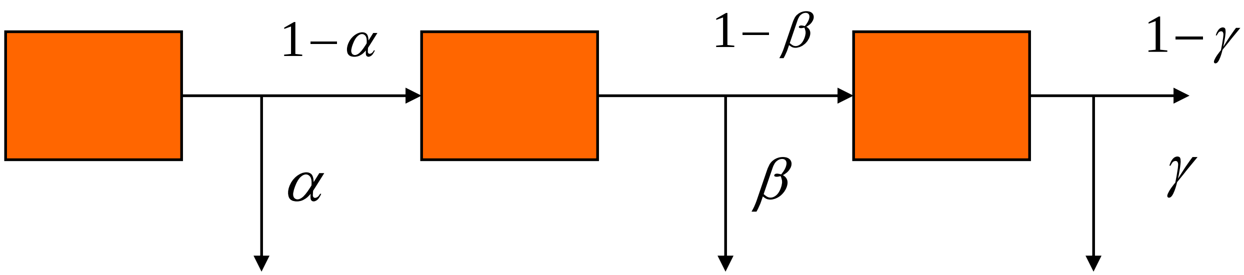
subject to

$$\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{a}_{\mathrm{ij}} X_{it} \leq c_{jt} + \mathrm{O}_{\mathrm{jt}} \qquad \text{for all } i,t$$

$$X_{it}, S_{it}, I_{it}^{+}, I_{it}^{-}, \mathrm{O}_{\mathrm{jt}} \geq 0 \qquad \text{for all } i,t$$

Aggregate Planning - Extensions

- **Yield loss**



$\alpha, \beta, \gamma \ldots$ fraction of output that is lost

$y_{ij} \ldots$ cumulative yield from station j onward

(including station j) for product i

we must release $\dfrac{d}{y_{ij}}$ units of i into station j

Aggregate Planning - Extensions

- **Basic model + Yield loss extension (no backorders)**

$$\max \sum_{t=1}^{\bar{t}} \sum_{i=1}^{m} (\mathrm{r_i} S_{it} - h_i I_{it})$$

subject to

$$\underline{d}_{it} \leq S_{it} \leq \bar{d}_{it} \qquad \text{for all } i,t$$

$$\sum_{i=1}^{m} \frac{\mathrm{a_{ij}} X_{it}}{y_{ij}} \leq c_{jt} \qquad \text{for all } j,t$$

$$I_{it} = I_{it-1} + X_{it} - S_{it} \qquad \text{for all } i,t$$

$$X_{it}, S_{it}, I_{it} \geq 0 \qquad \text{for all } i,t$$

Aggregate Planning - WorkforcePlanning

- **Single product, workforce resizing, overtime allocation**
- **Notation**

$$\mathrm{b} = \text{number of man-hours required to produce one unit of product}$$

$$\mathrm{l} = \text{cost of regular time in dollars/man-hour}$$

$$l' = \text{cost of overtime in dollars/man-hour}$$

$$\mathrm{e} = \text{cost to increase workforce by one man-hour per period}$$

$$\mathrm{e}' = \text{cost to decrease workforce by one man-hour per period}$$

$$\mathrm{W}_\mathrm{t} = \text{workforce in period t in man-hours of regular time}$$

$$\mathrm{H}_\mathrm{t} = \text{increase in workforce from period t-1 to t in man-hours}$$

$$\mathrm{F}_\mathrm{t} = \text{decrease in workforce from period t-1 to t in man-hours}$$

$$\mathrm{O}_\mathrm{t} = \text{overtime in period t in hours}$$

Aggregate Planning - WorkforcePlanning

- **LP formulation: maximize net profit, including labor, overtime, holding, and hiring/firing costs**
- **subject to constraints on sales, capacity,...**

$$\max \sum_{t=1}^{\bar{t}} \left\{ rS_t - h\, I_t - lW_t - l'O_t - eH_t - e'F_t \right\}$$

subject to

$d_t \leq S_t \leq \bar{d}_t$	for all t
$a_j X_t \leq c_{jt}$	for all j, t
$I_t = I_{t-1} + X_t - S_t$	for all t
$W_t = W_{t-1} + H_t - F_t$	for all t
$bX_t \leq W_t + O_t$	for all t
$X_{t,} S_{t,} I_t , O_{t,} W_{t,} F_{t,} H_{t,} \geq 0$	for all t

AP-WP Example

- **Revenue: 1000$**
- **worker capacity: 168h/month**
- **initially 15 workers**
- **no initial inventory**
- **holding costs: 10$/unit/month**
- **regular labor costs: 35$/hour**
- **overtime: 150% of regular**
- **hiring costs: 2500$ (2500/168 ~ 15$ per man-hour)**
- **lay-off costs: 1500$ (1500/168 ~ 9$ per man-hour)**
- **no backordering**
- **demands over 12 months:**

200, 220, 230, 300, 400, 450, 320, 180, 170,170, 160, 180

- **demands must be met! (S=D)**

AP-WP Example(cont.)

- **Determine over a 12 month horizon:**
  - **Number of workers: W**
  - **Output: X**
  - **Overtime use: O**
  - **Inventory: I**
  - **(H, F are additional choice variables in the model)**

Aggregate Planning - WorkforcePlanning

Parameters:													
r	1000												
h	10												
l	35												
l'	52,5												
e	15												
e'	9												
b	12												
I_0	0												
W_0	2520												
t	1	2	3	4	5	6	7	8	9	10	11	12	total
	200	220	230	300	400	450	320	180	170	170	160	180	2980

Desicion Variables:													
t	1	2	3	4	5	6	7	8	9	10	11	12	
Xt													
Wt													
Ht													
Ft													
It													
Ot													
Objective:													
Profit	2980000	$											

Aggregate Planning - WorkforcePlanning

Constraints:				
I1-I0-X1	0,00	=	-200	d_1
I2-I1-X2	0,00	=	-220	d_2
I3-I2-X3	0,00	=	-230	d_3
I4-I3-X4	0,00	=	-300	d_4
I5-I4-X5	0,00	=	-400	d_5
I6-I5-X6	0,00	=	-450	d_6
I7-I6-X7	0,00	=	-320	d_7
I8-I7-X8	0,00	=	-180	d_8
I9-I8-X9	0,00	=	-170	d_9
I10-I9-X10	0,00	=	-170	d_10
I11-I10-X11	0,00	=	-160	d_11
I12-I11-X12	0,00	=	-180	d_12
W1-W0-H1+F1	-2520,00	=	0	
W2-W1-H2+F2	0,00	=	0	
W3-W2-H3+F3	0,00	=	0	
W4-W3-H4+F4	0,00	=	0	
W5-W4-H5+F5	0,00	=	0	
W6-W5-H6+F6	0,00	=	0	
W7-W6-H7+F7	0,00	=	0	
W8-W7-H8+F8	0,00	=	0	
W9-W8-H9+F9	0,00	=	0	
W10-W9-H10+F10	0,00	=	0	
W11-W10-H11+F11	0,00	=	0	
W12-W11-H12+F12	0,00	=	0	

bX1-W1-O1	0,00	<=	0	
bX2-W2-O2	0,00	<=	0	
bX3-W3-O3	0,00	<=	0	
bX4-W4-O4	0,00	<=	0	
bX5-W5-O5	0,00	<=	0	
bX6-W6-O6	0,00	<=	0	
bX7-W7-O7	0,00	<=	0	
bX8-W8-O8	0,00	<=	0	
bX9-W9-O9	0,00	<=	0	
bX10-W10-O10	0,00	<=	0	
bX11-W11-O11	0,00	<=	0	
BX12-W12-O12	0,00	<=	0	

Aggregate Planning - WorkforcePlanning

Parameters:													
r	1000												
h	10												
l	35												
l'	52,5												
e	15												
e'	9												
b	12												
I_0	0												
W_0	2520												
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	total
	200	220	230	300	400	450	320	180	170	170	160	180	2980
Desicion Variables:													
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
Xt	302,86	302,86	302,86	302,86	302,86	302,86	302,86	180,00	170,00	170,00	170,00	170,00	2980,00
Wt	3634,29	3634,29	3634,29	3634,29	3634,29	3634,29	3634,29	2160,00	2040,00	2040,00	2040,00	2040,00	35760,00
Ht	1114,29	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1114,29
Ft	0,00	0,00	0,00	0,00	0,00	0,00	0,00	1474,29	120,00	0,00	0,00	0,00	1594,29
It	102,86	185,71	258,57	261,43	164,29	17,14	0,00	0,00	0,00	0,00	10,00	0,00	1000,00
Ot	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Objective:													
Profit	1687337	$											

Aggregate Planning-Summary

<u>The following scenarios have been discussed:</u>

- **single product, single resource, single process**
  find: workforce, output, inventory (w. or w/o backorders)

- **multiple products, single resource, single process**
  find: workforce, all outputs, all inventories (w. or w/o backorders)

- **multiple products, multiple resources, multiple processes (workforce given)**
  find: all outputs, all inventories, use of processes

Aggregate Planning-Summary

The following scenarios have been discussed:

- **multiple products, multiple workstations (workstation capcities given)**
  find: all sales, all outputs, all inventories (w. or w/o backorders)
- **multiple products, multiple workstations**
  find: all sales, all outputs, all inventories (w. or w/o backorders), OT
- **single product, multiple workstations, one resource**
  find: workforce, all sales, all outputs, all inventories (w. or w/o backorders), OT

Aggregate Planning

- **Work to do:**

- **Examples: 5.7, 5.8abcdef, 5.9abcd, 5.10abcd, 5.16abcd, 5.21, 5.22, 5.29, 5.30**

  Replace capacity columns of table in problem 5.29 with

Month	Machine	Worker
1	1350	19000
2	1270	19000
3	1350	19500

- **Minicase BF SWING II**