LP-Model for Job Shop Problems

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LP-Model: Variables

Given Data:

$n \ldots$ number of jobs

$m \ldots$ number of machines

$D_i \ldots$ due date of job i

$p_{ij} \ldots$ processing time of job i on machine j

$seq_{ij} \ldots$ position in sequence of job i on machine j

Decision Variables:

$$C_{max} = \max\{C_i\}$$

$C_i \ldots$ completion time of job i

$T_i \ldots$ tardiness of job i

$$\delta_i = \begin{cases} 1 & \text{if } T_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

$start_{ij} \ldots$ start of job i on machine j

$$y_{ij}^k = \begin{cases} 1 & \text{if job i follows job j on machine k} \\ 0 & \text{otherwise} \end{cases}$$

LP-Model: Objective

Objective 1: $\min C_{max} \cdot M + \sum_{i}^{n} T_i \cdot 1/M + \sum_{i}^{n} \delta_i \cdot 1/M + \sum_{i}^{n} C_i \cdot 1/M$

Objective 2: $\min C_{max} \cdot 1/M + \sum_{i}^{n} T_i \cdot M + \sum_{i}^{n} \delta_i \cdot 1/M + \sum_{i}^{n} C_i \cdot 1/M$

Objective 3: $\min C_{max} \cdot 1/M + \sum_{i}^{n} T_i \cdot 1/M + \sum_{i}^{n} \delta_i \cdot M + \sum_{i}^{n} C_i \cdot 1/M$

Objective 4: $\min C_{max} \cdot 1/M + \sum_{i}^{n} T_i \cdot 1/M + \sum_{i}^{n} \delta_i \cdot 1/M + \sum_{i}^{n} C_i \cdot M$

$$C_{max} \geq start_{i,seq_{i,m}} + p_{i,seq_{i,m}} \qquad \forall i = 1, \ldots, n$$

$$C_i \geq start_{i,seq_{i,m}} + p_{i,seq_{i,m}} \qquad \forall i = 1, \ldots, n$$

LP-Model: Constraints

Precedence constraints

$$start_{i,seq_{i,j+1}} \geq start_{i,seq_{i,j}} + p_{i,seq_{i,j}} \qquad i = 1,\dots,n \quad j = 1,\dots,m-1$$

Disjunction Constraints

$$start_{ik} + p_{ik} \leq start_{jk} + M \cdot y_{ij}^k \qquad k = 1,\dots,m \quad i,j = 1,\dots,n \wedge i < j$$

$$start_{jk} + p_{jk} \leq start_{ik} + M \cdot (1 - y_{ij}^k) \qquad k = 1,\dots,m \quad i,j = 1,\dots,n \wedge i < j$$

Bounds on Tardiness

$$T_i \geq start_{i,seq_{i,m}} + p_{i,seq_{i,m}} - D_i \qquad i = 1,\dots,n$$

$$T_i \leq \delta_i \cdot M$$

$\delta_i \leq T_i \cdot M$ not needed

Non-negativity Constraints

$$T_i \geq 0 \qquad i = 1,\dots,n$$

$$C_i \geq 0 \qquad i = 1,\dots,n$$

$$T_i \geq 0 \qquad i = 1,\dots,n$$

$$start_{ij} \geq 0 \qquad i = 1,\dots,n \quad j = 1,\dots,m$$

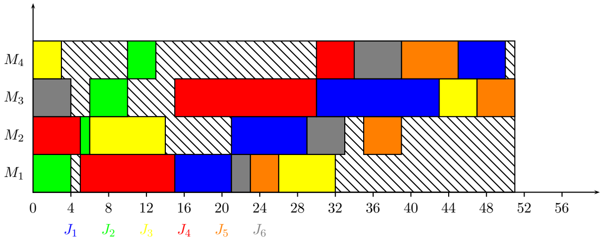

Figure: Objective C_{max}

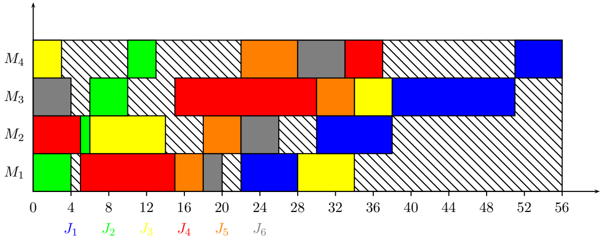

Figure: Objective Total Tardiness

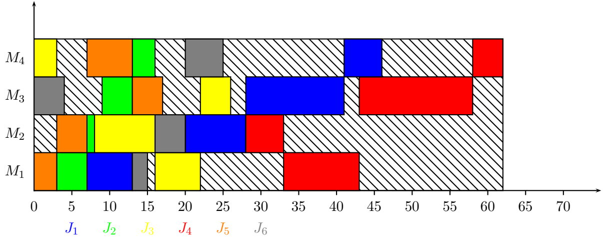

Figure: Objective Number of Tardy Jobs

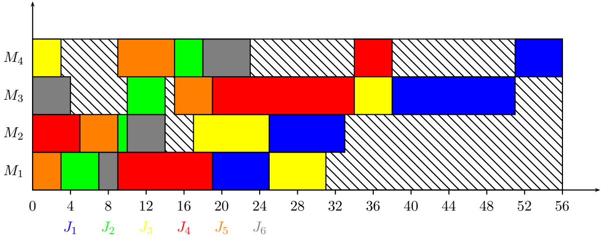

Figure: Objective Total Flowtime