MIP for the Lot-sizing in MRP

Indices:

P	Number of products in BOM
T	Planning horizon
M	Number of resources
i	Label of each item in BOM assumed that all labels are sorted with respect to their low level code
t	Specified Period t in T
m	Specified Resource m in M

Parameters

$\Gamma(i)$	Set of immediate successors of item *i*
$\Gamma^{-1}(i)$	Set of immediate predecessors of item *i*
s_i	Setup cost for item *i* (assumed to be constant over time horizon)
c_{ij}	Quantity of item *i* required to produce one unit of item *j*.
h_i	Holding cost for item *i* (assumed to be constant over time horizon)
a_{mi}	Capacity needed on resource *m* for one unit of item *i*
b_{mi}	Setup time for item *i* on resource *m*
L_{mt}	Available capacity of resource *m* in period *t*
oc_m	Overtime cost of resource *m*
G	Big number in this case assume G= 500000
$D_{it}=$	$\begin{cases} \text{External demand for product i in period t} & \text{if i is finished items} \\ 0 & \text{otherwise} \end{cases}$

Variables

x_{it}	Delivered quantity of item *i* at the beginning of period *t*.
I_{it}	Inventory level of item *i* at the end of period *t*.
O_{mt}	Overtime hours required for machine *m* in period *t*
$y_{it} =$	$\begin{cases} 1 & \text{when item } i \text{ is produced in period } t \\ 0 & \text{otherwise} \end{cases}$

For simplicity, we assume that the demand $d_{i,t}$ for all end-products is given. The problem can then be formulated as a mixed integer program:

$$\min \sum_{i=1}^{P}\sum_{t=1}^{T}(s_i y_{it} + h_i I_{it}) + \sum_{t=1}^{T}\sum_{m=1}^{M} oc_m O_{mt} \tag{1}$$

subject to the set of constraints

$$I_{i,t} = I_{i,t-1} + x_{i,t} - \sum_{j\in\Gamma(i)} c_{ij} x_{jt} - D_{it} \qquad \forall i,t \tag{2}$$

$$\sum_{i=1}^{P}(a_{mi}x_{it} + b_{mi}y_{it}) \le L_{mt} + O_{mt} \qquad \forall m,t \tag{3}$$

$$x_{it} - Gy_{it} \le 0 \qquad \forall i,t \tag{4}$$

$$I_{it} \ge 0,\quad x_{it} \ge 0,\ y_{it} \in \{0,1\} \qquad \forall i,t \tag{5}$$