The Dynamics of a Simple Relative Adjustment-Cost Framework

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This paper considers a capital accumulation model with the specific feature that adjustment costs depend on investment relative to the size of the capital stock. This framework has, beyond its plausible yet neglected setting, a number of interesting consequences. In particular, the possibility of multiple equilibria, of an unstable steady state and thus of a ('history dependent') threshold associated with concavity is surprising given a voluminous literature on multiple, history dependent equilibria emphasising non-concavities (or convexities).

JEL #:C61, C62

1. Introduction

This paper investigates the general stability properties when agents face adjustment costs that depend on the relative rather than the absolute change. While the standard adjustment-cost framework has led to a substantial amount of applications since the works of Eisner and Strotz (1963) and Lucas (1967), a number of papers have used costs in relative adjustments, e.g. Hayashi (1982), D'Autume and Michel (1985) and Jorgensen and Kort (1993), but have failed to look at the general dynamic properties due to this particular specification. This paper attempts to fill this gap. Adjustment costs depending on investment relative to capital stock implies that these costs are low when the capital stock is large. This can be illustrated by, e.g., the fact that when installing a new machine a small firm has to stop production completely while a large firm is more flexible because there production can continue on a parallel production line. Another motivation for taking into account relative adjustment costs can be learning by doing in the installation process: if capital stock is large, a lot of machines have been installed in the past so that this firm has a lot of experience, implying that it is more efficient in installing new machines. For other motivations concerning the validity of relative adjustment costs we refer to Lucas (1967). Moreover, this relative adjustment cost framework is very suitable to highlight some of the points that are overlooked in the literature dealing with multiple equilibria, thresholds and history dependence.

The paper is organised as follows: the presentation of the basic relative-adjustment cost framework is followed by a brief review of the standard adjustment cost approach, and the analysis of the framework. Concluding remarks complete this study.

2. Framework

As a starting point, consider the simple static choice where $U(x)$ denotes an agent's objective that is twice continuously differentiable and concave. Assuming that the agent controls x directly implies that the benefit maximising agent chooses at each instant of time t

$$x^0 := \arg\max U(x). \tag{1}$$

This static choice x^0 is assumed to be unique and in the 'interior', $U'(x^0) = 0$. However, we will assume for the remainder of the paper that the agent cannot control x directly but only the (gross) change or the accumulation of x. Of course, stocks like capital, capacity etc. require corresponding investments. Furthermore many so-called flows or controls cannot be altered instantaneously and are the results of sluggish adjustments. Examples abound, production, budget deficits, etc. While adjustment costs - costs depending on the absolute size of the adjustment - figure prominently in the literature, the emphasis of this investigation is on relative adjustment, where relative is based on the existing stock. That is, what matters is not so much how many machines are replaced but rather the share of machines that are replaced, added or scrapped. More precisely, we consider the following adjustment cost problem:

$$\max_{\{u(t)\}} \int_0^\infty \exp(-rt)[U(x(t)) - C(u(t)/x(t))]\mathrm{d}t, \tag{2}$$

$$\dot{x}(t) = u(t) - \delta x(t), \quad x(0) = x_0. \tag{3}$$

The stock x generates a concave payoff U and C denotes the costs contingent on the share of capital expansion. As in the traditional adjustment cost literature, C is a scalar function that is increasing, convex and satisfies $C(0) = C'(0) = 0$. An important consequence of relative instead of absolute adjustment costs is that this cost function C is not jointly convex, although it is convex in u and x separately. The economic source of the non-convexity of C are the increasing returns to scale and the mathematical reason is that the determinant of the Hessian of $C(u/x)$ equals $-(C'/x^2)^2$ and is thus negative. As a consequence, the objective ($U - C$) and thus the associated Hamiltonian need not be jointly concave in particular around 'small' steady states. This non-concavity coupled with concave domains (x sufficiently large) can lead to multiple equilibria including unstable ones. Yet one of the interesting results of our analysis of adjustment costs in relative rather than absolute terms is that neither the existence of multiple equilibria, nor the potential instability and the associated threshold property require this non-concavity.

Defining the current value Hamiltonian H using λ to denote the costate of x,

$$H = U(x) - C(u/x) + \lambda(u - \delta x), \tag{4}$$

we obtain the following first order conditions for interior solutions of the optimal control problem (2) – (3):

$$H_u = -C'/x + \lambda = 0, \tag{5}$$

$$\dot{\lambda} = (r + \delta)\lambda - U' - C'u/x^2. \tag{6}$$

The second order necessary optimality condition, $H_{uu} \leq 0$, the so-called Legendre-Clebsch condition, is satisfied due to the assumed convexity of C, $H_{uu} = -C''/x^2$. These necessary optimality conditions are sufficient if the Hamiltonian is concave, either in (u, x) or in the Mangasarian version, the maximised Hamiltonian in x, but not otherwise. Since the analysis is restricted to the necessary conditions, as is typical for the economics literature, a little vagueness remains on the optimality of the paths characterised in the non-concave domain. This ambiguity can be relaxed at least a little bit, since an optimum must exist for a compact space, and this optimum must of course satisfy the necessary optimality conditions that characterise a unique solution due to $H_{uu} < 0$.

In addition to the static decision x^0 and the intertemporal decision satisfying (5) and (6), one may consider the stationary choice, i.e. maximising the integrand of the objective with respect to the capital stock x accounting for the corresponding stationary investment, $u = \delta x$.

$$\max_{x} U(x) - C(\delta) \tag{7}$$

Hence, the optimal stationary choice in (7) coincides with the static choice x^0, because each arbitrarily selected state x will require the adjustment $u = \delta x$, i.e. the same relative adjustment of the magnitude δ so that the corresponding adjustment costs are independent of x. This result is in contrast to the stationary analysis of the conventional adjustment cost framework that is briefly reviewed in the following section.

3. Brief review of standard adjustment cost models

In order to highlight the consequences arising from the consideration of relative instead of absolute adjustment costs, we review the familiar adjustment cost framework which differs from (2) and (3) only in the argument of the adjustment costs, $C = C(u)$. The corresponding optimal policy must satisfy the following first order conditions (analogous to (4) – (6)):

$$-C' + \lambda = 0, \tag{8}$$

$$\dot{\lambda} = (r + \delta)\lambda - U'. \tag{9}$$

Eliminating the control yields for the state differential equation $\dot{x}(\mathrm{t}) = C'^{-1}(\lambda) - \delta x(t)$ which coupled with (9) forms the canonical equations with the associated Jacobean and its determinant:

$$J = \begin{pmatrix} -\delta & \dfrac{1}{C''} \\ -U'' & r+\delta \end{pmatrix} \Rightarrow \det(J) = -\delta(r + \delta) + U''/C'' < 0. \qquad (10)$$

The canonical equations system has a unique saddle point equilibrium since det(J) is negative. The corresponding phase portrait is shown in Fig. 1. In short: Amending a conventional static optimisation problem for the dynamics resulting from adjustments (or investments) ensures a unique and stable long run equilibrium. This equilibrium is given by the intuitive condition that the marginal instantaneous benefit equals the corresponding marginal costs of maintaining this stock, including interest costs,

$$U'(\delta x) = (r + \delta)C'(\delta x). \qquad (11)$$

The stationary analysis of this standard adjustment cost framework, $[U(x) - C(\delta x)] \rightarrow \max$, yields $U' = \delta C'$. Hence, dynamic considerations modify the above stationary analysis only by the inclusion of the interest costs in (11) and by a dynamic transition process for starting out of equilibrium of the kind shown in Fig. 1; for comparison, all three levels of capital stocks - the long run optimum (or steady state, x_∞), the one implied by a stationary instead of a truly dynamic analysis and the static choice x^0 - are included.

Insert Fig. 1 here

4. Stability Analysis

Using the maximum principle (5) to eliminate the control in (3) and (6) and introducing $\sigma = (u/x)C''/C'$, the elasticity of marginal costs, yields

$$u^* = u(x, \lambda), \qquad (12.1)$$

$$u_x = [(u/x)C''+C']/C'' = (u/x)(1 + \sigma)/\sigma, \text{ thus } u_x = \delta(1 + \sigma)/\sigma \text{ at a steady state} \qquad (12.2)$$

$$u_\lambda = x^2/C''. \qquad (12.3)$$

Substituting $u(x, \lambda)$ from (12) into (3) and (6), yields the canonical equations in (x, λ)

$$\dot{x} = u(x, \lambda) - \delta x, \qquad (13)$$

$$\dot{\lambda} = (r + \delta)\lambda - U'(x) - u(x, \lambda)C'(u(x, \lambda)/x)/x^2. \qquad (14)$$

In order to study the long run but intertemporal optimal choice and its deviation from the stationary one, set $\dot{\lambda}$ in (14) equal to zero, while substituting from the maximum principle, $\lambda = C'/x$ and the steady state requirement $u = \delta x$ (see (13)). This yields $U'(x) = r\lambda$, and since $\lambda >$

0 for any $x > 0$ due to (5), *the long run dynamic outcome falls short of the static choice* x^0 that satisfies $U' = 0$.

The Jacobian J associated with the canonical equations (13) and (14) and evaluated at a steady state (thus $u/x = \delta$ and using the elasticity σ) and its determinant are given by:

$$J = \begin{pmatrix} \dfrac{\delta}{\sigma} & \dfrac{x^2}{C''} \\ -\dfrac{C'^2}{C'' x^2} - U'' & r - \dfrac{\delta}{\sigma} \end{pmatrix} \Rightarrow \det(J) = \frac{r\delta}{\sigma} + \frac{x^2 U''}{C''}. \tag{15}$$

Observe that the term in the first row and first column of J is positive despite the depreciation. That is, $\dfrac{\partial f}{\partial x} = -\delta < 0$ but $\dfrac{\partial \dot{x}}{\partial x} = \dfrac{\partial f}{\partial x} + \dfrac{\partial u}{\partial x} = \dfrac{\delta}{\sigma} > 0$ at a steady state and this positive derivative can lead to complexities, compare Wirl and Feichtinger (1999) for a general analysis of concave control problems. The last term of det(J) given in (15) is definitely negative due to the concavity of U and the convexity of C. *Therefore, ever-lasting equipment, $\delta = 0$, allows only for a stable steady state*. Increasing either r or δ raises det(J) and thus the chances for an unstable steady state, and on the other hand increasing σ reduces det(J) and hence fosters stability.

In the remainder of the paper, the analysis is restricted to quadratic adjustment costs,

$$C(z) = ½cz^2 = ½c(u/x)^2 \;\Rightarrow\; \sigma = 1 \text{ so that } \det(J) = r\delta + \frac{x^2 U''}{C''}, \tag{16}$$

and a linear-quadratic payoff function,

$$U(x) = x - ½x^2, \tag{17}$$

because these simple specifications are sufficient to generate the complexities of interest in this paper and allow for analytical solutions. The profit in (17) is w.l.i.g. normalised so that $x^0 = 1$ and marginal benefit is non-negative for $x \in [0, 1]$.

Given these specifications in (16) and (17), the corresponding optimal investment policy is

$$u^* = \lambda x^2/c, \tag{18}$$

which in turn simplifies the canonical equations to:

$$\dot{x} = x(\lambda x/c - \delta), \tag{19}$$

$$\dot{\lambda} = (r + \delta)\lambda - (1 - x) - \lambda^2 x/c. \tag{20}$$

This system (19) and (20) has three pairs of steady states that are reported below in ascending order of the value of the state x:

$$\begin{aligned} &(0,\ 1/(r + \delta)), \\ (x_\infty, \lambda_\infty) = &(\tfrac{1}{2}(1 - \sqrt{D}),\ \tfrac{1}{2}(1 + \sqrt{D})/r), \\ &(\tfrac{1}{2}(1 + \sqrt{D}),\ \tfrac{1}{2}(1 - \sqrt{D})/r). \end{aligned} \tag{21}$$

The discriminant in (21) must be non-negative,

$$D = 1 - 4cr\delta \geq 0, \tag{22}$$

because otherwise only the first and trivial steady state, $x_\infty = 0$, exists. Why do we have three steady states (for $\delta > 0$)? Despite this apparently simple framework, the familiar phase diagram analysis is a little bit tricky. First, we have two $\dot{x} = 0$ isoclines in (x, λ) plane:

$$\dot{x} = 0 \text{ if either} \begin{cases} \mathrm{x} = 0 \\ \quad \text{or} \\ \lambda = c\delta/x \end{cases} \tag{23}$$

and possibly three $\dot{\lambda} = 0$ isoclines (examples will be drawn below):

$$\dot{\lambda} = 0 \text{ if } \lambda = \begin{cases} \dfrac{1}{r+\delta} \\ \dfrac{\mathrm{c(r}+\delta) \pm \sqrt{\mathrm{c}^2(\mathrm{r}+\delta)^2 + 4cx(x-1)}}{2x} \end{cases} \text{for } x \begin{array}{c} = 0 \\ > 0 \end{array} \text{ and } x \in [0,\ 1] \tag{24}$$

The domain of the $\dot{\lambda} = 0$ isocline is restricted to states where $c(r + \delta)^2 > 4x(1 - x)$, which consequently restricts the set of feasible steady states to either 'low' or 'high' levels unless $c(r + \delta)^2 > 1 = \max\{4x(1 - x),\ x \in [0, 1]\}$. Fig. 2 draws attention to the restrictions concerning the $\dot{\lambda} = 0$ isocline.

Insert Fig. 2 here

The Jacobean (15) simplifies in the case of quadratic costs C and linear-quadratic utilities U to

$$J = \begin{pmatrix} \delta & \dfrac{x^2}{c} \\ \dfrac{x^2 - c\delta^2}{x^2} & r - \delta \end{pmatrix} \text{ at a steady state.} \tag{25}$$

The determinant of the Jacobean at the three different steady states is:

$$\det(J) = \begin{cases} x = 0 : -\delta(r+\delta) < 0, \text{ at the smallest steady state,} \\ x > 0 : \delta r - \dfrac{x^2}{c} = \begin{cases} \dfrac{\sqrt{D} - D}{2c} > 0 \text{ at the intermediate and} \\ -\dfrac{\sqrt{D} + D}{2c} < 0 \text{ at the largest steady state,} \end{cases} \end{cases} \tag{26}$$

after substituting the stationary solutions (21); the claimed signs follow from $D < 1$. Since $\det(J) < 0$ at $x = 0$ and for the largest of the steady states, these two steady states are (locally) stable while the intermediate is unstable ($\det(J) > 0$).

The determinant of the Hessian of the Hamiltonian equals at the two positive steady states,

$$\det\begin{pmatrix} H_{uu} & H_{ux} \\ H_{ux} & H_{xx} \end{pmatrix} = \frac{c}{x^4}(x^2 - c\delta^2) = \frac{8c\left[1 - 2c\delta(r+\delta) \pm \sqrt{D}\right]}{\left(1 \pm \sqrt{D}\right)^4} \tag{27}$$

where the '+' before the root in (27) corresponds to the largest steady state value of x ($= \frac{1}{2}(1 + \sqrt{D})$). Hence, this determinant in (27) will be positive and thus the Hamiltonian is concave (the main diagonal elements in (27) are both negative) for any steady state $x > \delta\sqrt{c}$, even if the determinant of the Jacobean is positive; $x < \sqrt{r\delta c}$. Indeed, such examples coupling instability and concavity are easy to construct if the necessary requisite, $r > \delta > 0$, is met. Therefore fast depreciation, $r < \delta$, implies that the unstable steady state will always be in the non-concave domain. Hence, the condition of a long lasting equipment, $\delta < r$, is necessary but not sufficient to facilitate instability coupled with concavity. Yet varying e.g. c allows for a continuous transition of this unstable steady state from the concave into the non-concave domain. This observation highlights that *violating concavity is not essential for an instability*.

Insert Fig. 3 here

An example where all positive steady states (and the associated stationary controls) rest in the concave domain is shown in Fig. 3. In this example, which is characterised by long lasting (but not everlasting) equipment and relatively high discount rates, only the solution of the isocline with the negative of the root in (24) is relevant (the 'upper' solution exceeds $\dot{x} = 0$

everywhere) and the support of $\dot{\lambda} = 0$ is unrestricted, since $c(r + \delta)^2 > 1$. However, the same steady state values f-or x result from (21) if we just revert the values between r and δ and thus consider a process of a rather short lived equipment, where the static consideration could be expected to provide a reasonable proxy. Despite the same steady states, both positive steady states (and not only the unstable one) are in the domain where the Hamiltonian is non-concave. *This highlights that the properties 'stable' or 'unstable' are decoupled from the characterisation concave or non-concave*. Furthermore, the phase diagram differs from the one in Fig. 3 because then the other solution of $\dot{\lambda} = 0$ (more precisely the 'larger' one, i.e., the one with the '+' in front of the root in (24)) intersects $\dot{x} = 0$.

As already indicated above, decreasing the value of c, the lower of the two positive steady states moves from the non-concave into the concave domain at $c = 1/(\delta + r)^2$, if $r > \delta$ Interestingly, the determinant of the Hessian matrix of the Hamiltonian vanishes at that point where the support of the costate isocline stops being global (coming from the 'right'). *That is for slow depreciation, $r > \delta$, demanding that $\dot{\lambda} = 0$ is defined for all feasible states, $x \in [0, 1]$, is equivalent to the fact that all positive steady states are in the concave domain.* Consequently reducing in the example of Fig. 3 the cost parameter from c = 1½ to c = 0.826446... produces this boundary case where the lower of the positive steady states is at the boundary of the concave domain in the (x, u) plane. Reducing c further implies (a) that solutions for $\dot{\lambda} = 0$ do not exist in the 'middle' of [0, 1], and (b) that the lower of the positive steady states will fall into the non-concave domain. Properties of these kinds of unstable, steady states in non-concave domains have been theoretically analysed in Skiba (1978) and Dechert and Nishimura (1983), which in turn triggered many applications with an emphasis on explaining different growth and development patterns. An important economic implication of Fig. 3 is the following. Induced by the fact that the Hamiltonian is concave, a characteristic of the solution presented in Fig.3 is that λ, and thus the control variable u, is continuous in the state variable x. This is an important distinction from the Skiba-Dechert-Nishimura framework in which $\lambda(x)$, and thus $u(x)$, is discontinuous at a critical value typically close to the unstable steady state.

The analysis of the case where the domain of $\dot{\lambda} = 0$ and consequently the set of steady states is restricted, is complicated by this very fact of a reduced support of the costate isocline. Of course we know already from above that the unstable steady state must now lie in the non-concave domain. Interestingly, *'fast' depreciation, $r < \delta$, coupled with a restricted domain of*

$\dot{\lambda}$ = 0, i.e., $c(r + \delta)^2 > 4x(1 - x)$ for some x, implies that the larger of the positive steady states is in the concave domain. Conversely, a global support of $\dot{\lambda}$ = 0 $\forall x \in [0,1]$ implies that both positive steady states are associated with a non-concave Hamiltonian. An example where $r < \delta$ and the domain of $\dot{\lambda} = 0$ is constrained is shown in Fig. 4. We have the familiar three steady states, but a large domain where $\dot{\lambda} = 0$ does not exist at all. From $r < \delta$ follows that the left hand but positive steady state is unstable and within the non-concave domain and the partial non-existence of $\dot{\lambda} = 0$ coupled with $r < \delta$ implies that the Hamiltonian is concave at the positive and stable steady state.

Insert Fig. 4 here

Each steady state in the concave domain must be a node (including the unstable ones), compare Wirl and Feichtinger (1999), and see the example in Fig. 1. Yet a number of papers creates the opposite impression to a casual reader namely that an instability coupled with a non-concavity implies an unstable focus. For example, the unstable steady state in Fig. 4 is a focus too, but this property - non-concavity coupled with instability implies a focus – does not hold in general. In fact, the property (node or focus) of an unstable steady state depends in this non-concave case on subtle issues such as the slopes of the isoclines that are typically not checked in the literature. The specification of U and C allows obtaining the precise condition when an unstable steady state in the non-concave domain must be a node. Non-concavity requires that the determinant of the Hessian is negative, which is equivalent to:

$$c\delta(r - \delta) < \tfrac{1}{2}(1 - \sqrt{D})\sqrt{D}, \tag{28}$$

where the right hand side of (28) equals the steady state value of x times $\sqrt{D}$. The supposition of a node requires that the eigenvalues e_{12} at the unstable steady state

$$e_{12} = \frac{1}{2}\left[r \pm \sqrt{\frac{cr^2 + 2\left(D - \sqrt{D}\right)}{c}}\right] \tag{29}$$

are real, which is equivalent to:

$$D + \tfrac{1}{2}cr^2 > \sqrt{D}. \tag{30}$$

Squaring (30), which is permitted because the left hand side is positive assuming the existence of positive steady states (thus $D = (1 - 4cr\delta) > 0$), implies

$$1 - 4cr\delta < [(1 - 4cr\delta) + \tfrac{1}{2}r^2]^2 \tag{31}$$

from which one obtains immediately the following condition for the unstable steady state being a node:

$$\delta(r - \delta) < \tfrac{1}{4}r^2. \qquad (32)$$

Fig. 5 plots, for $c = 1$, the different domains of the unstable steady state with respect to the remaining model parameters δ and r. *The feasible domain can be divided into two parts, spirals and nodes, of which the latter set consists of concave and non-concave subsets. Moreover, the parts ignored in the literature on multiple equilibria – concave and nodes in the non-concave domain – cover a significant portion of the parameter space.*

Insert Fig. 5 here

5. Conclusions

While the conventional adjustment-cost framework alters little compared to the stationary outcome (just add interest costs), the relative adjustment cost framework has substantial implications:

1. The long run, intertemporal investment rule differs substantially from the corresponding stationary analysis (which in turn reproduces the static outcome ignoring adjustment costs entirely), because these adjustment costs depend only on the rate of depreciation and are thus independent of the state. Moreover, the intertemporally optimal steady states fall below the stationary outcome, which is to some extent surprising because the stationary analysis ignores the increasing returns (source of the non-concavity) associated with costs depending on relative adjustments.

2. In case of quadratic adjustment costs and benefits, multiple (more precisely three) steady states result of which one is unstable. This unstable steady state leads to a threshold (not necessarily identical to this unstable steady state): the stock should be run down for starting below the threshold, otherwise a high level should be attained. However, the properties of stability and instability of a steady state are unrelated to whether the optimisation problem is concave or non-concave.

3. In particular for slow depreciation processes, $r > \delta$, both positive (unstable and stable) steady states may fall into the concave domain. Incidentally requiring that both positive (unstable and stable) steady states are in the concave domain is equivalent to the condition that the domain of the costate-, $\dot{\lambda} = 0$, isocline is not restricted.

4. For 'fast' depreciation processes, $\delta > r$, the unstable steady state is always in the non-concave domain. However, again non-concavity is not suitable to discriminate between the different steady states, because the stable steady state may fall into the non-concave domain too (of course raising the issue, whether the first order conditions are sufficient too). Now, a restricted support of $\dot{\lambda} = 0$ is equivalent to a concave Hamiltonian at the larger of the positive steady states.

5. While an unstable focus is impossible within the concave domain, it is not necessary for an unstable steady state within the non-concave domain, i.e., the unstable steady state may be a node. This observation strengthens the overall result of this paper that the differences between concave and non-concave are subtler than the existing literature suggests.

6. From a policy point of view it is important to distinguish an unstable focus and an unstable node. The reason is that in the case of a node investment can be a continuous function of the capital stock level, while the implication of a focus is that the policy function is always discontinuous. In fact, the policy function is definitely continuous in case the node lies in the concave domain of the Hamiltonian (see Fig. 3).

Summing up, despite an extensive literature dealing with unstable steady states, the presented relative-adjustment cost framework is suitable to derive so far important but overlooked properties that concavity can induce history dependent evolutions, and that the unstable steady state can be a node; and as Fig. 5 highlights, these cases are far from being insignificant.

Aside from the theoretical contribution of uncovering dynamic properties overlooked in the corresponding literature on multiple equilibria, hysteresis, history dependence and alike, this paper has significant economic implications too. First, to encourage the use of the relative adjustment cost framework whenever costs depending on relative magnitudes seem significant. Second, the reasons for history dependent evolutions go beyond the familiar increasing returns to scale hypothesis, which allows to explain such phenomenon where the increasing returns to scale hypothesis fails. Third, it warns that the familiarly invoked discontinuity due to multiple equilibria and a local instability need not hold, and that an instability in the non-concave domain requires a check of the eigenvalues. This was not done so far so that many of the spirals shown in the phase diagrams in the literature need not exist, at least not in general. The related issue of the local dynamics of an unstable node in the non-concave domain requires further theoretical analysis, which should be the subject of future research.

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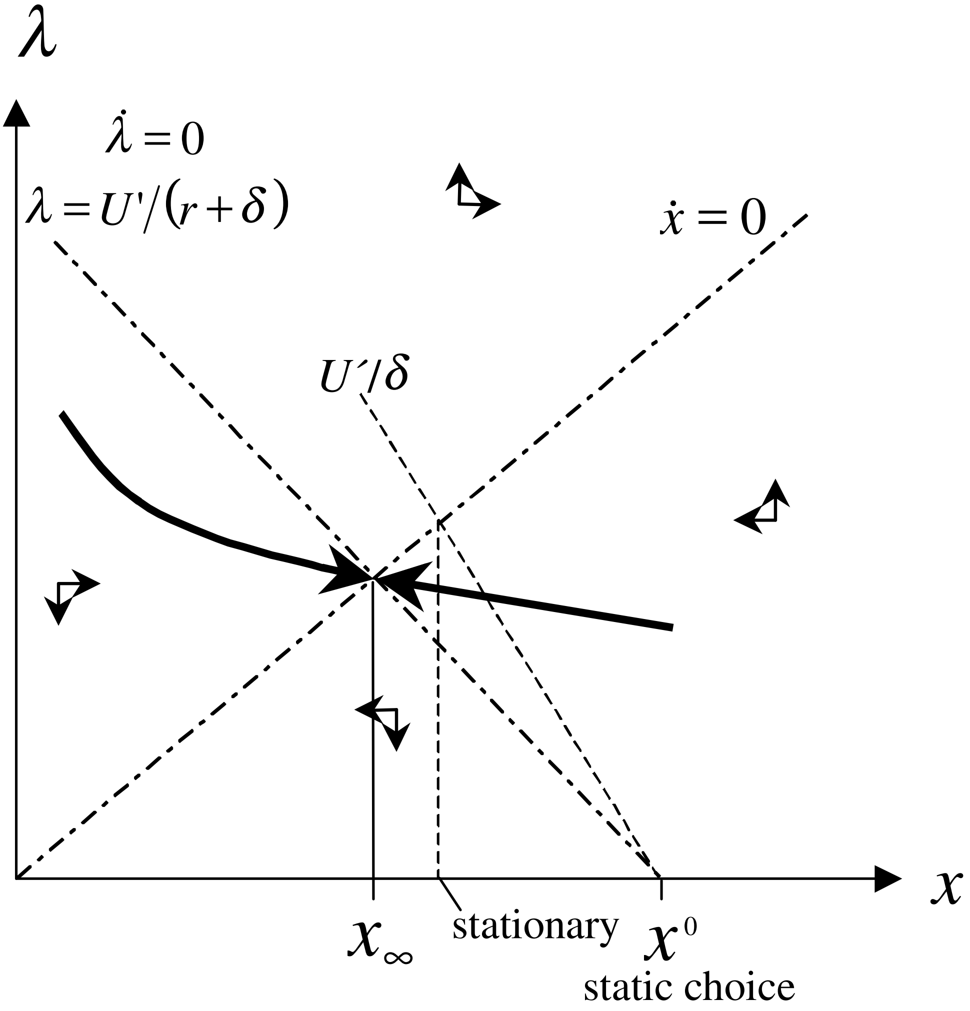
Fig. 4: Example, $c = 1$, r -= ¼ , $\delta = ½$, where the set of steady states (and the domain of $\dot{\lambda} = 0$), is restricted.

Fig. 5: Properties of an unstable steady state (for $c = 1$).



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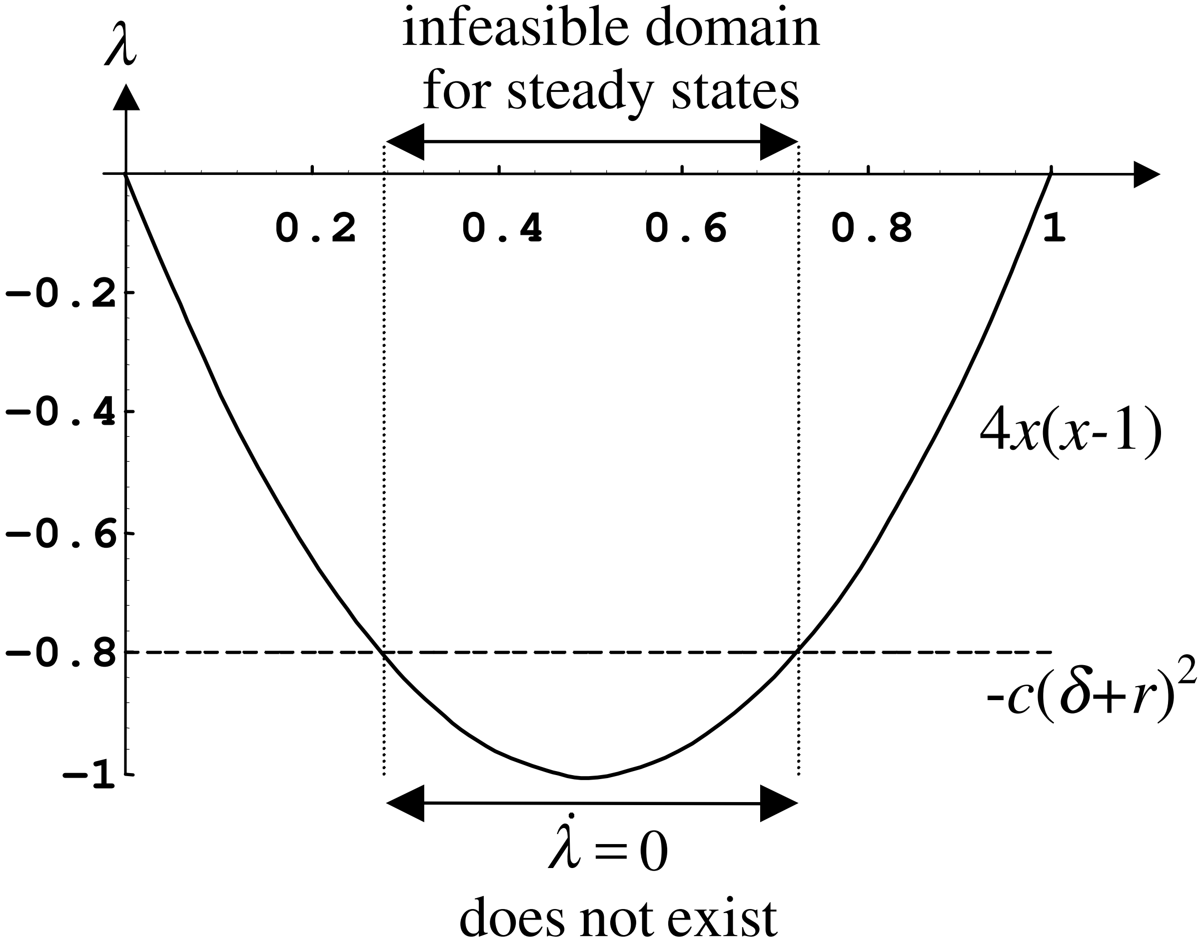
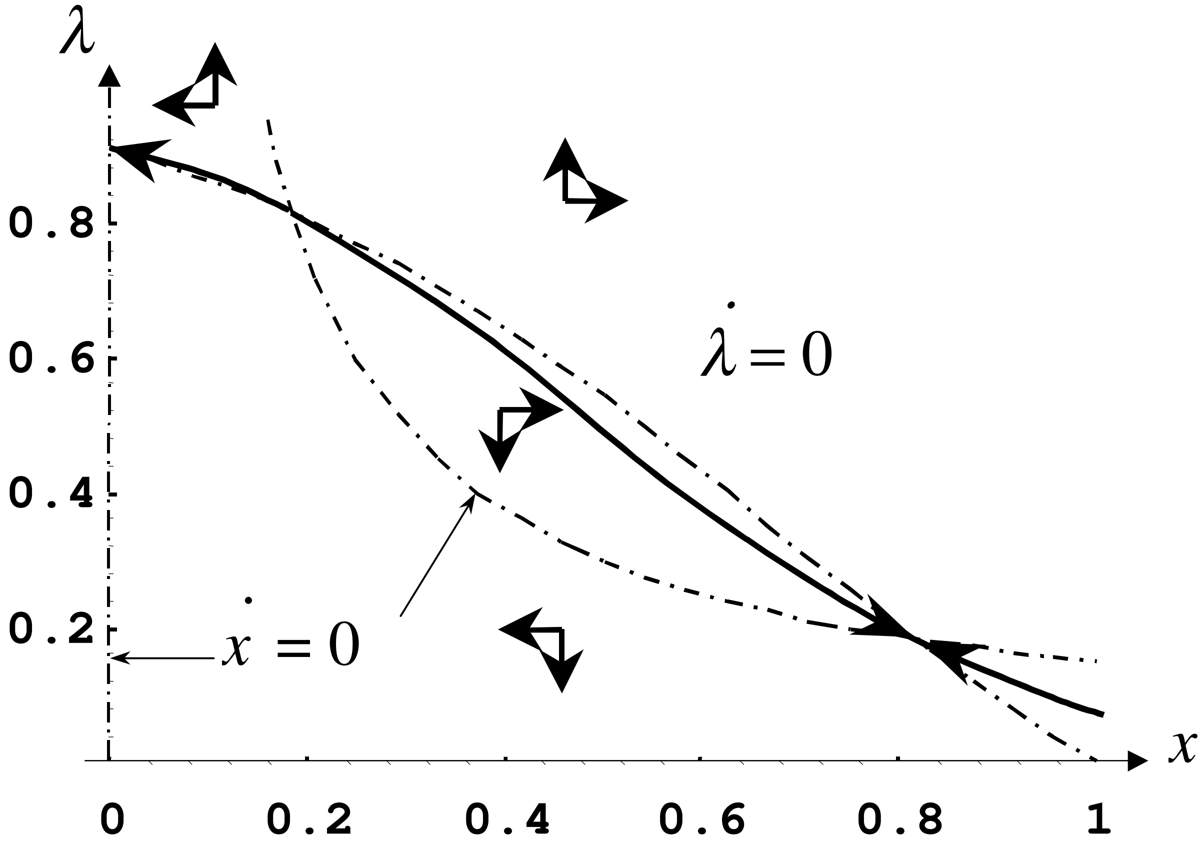


Fig. 2: Feasibility depending on the elements of the discriminant of the isocline $\dot{\lambda}$ = 0 in (24).



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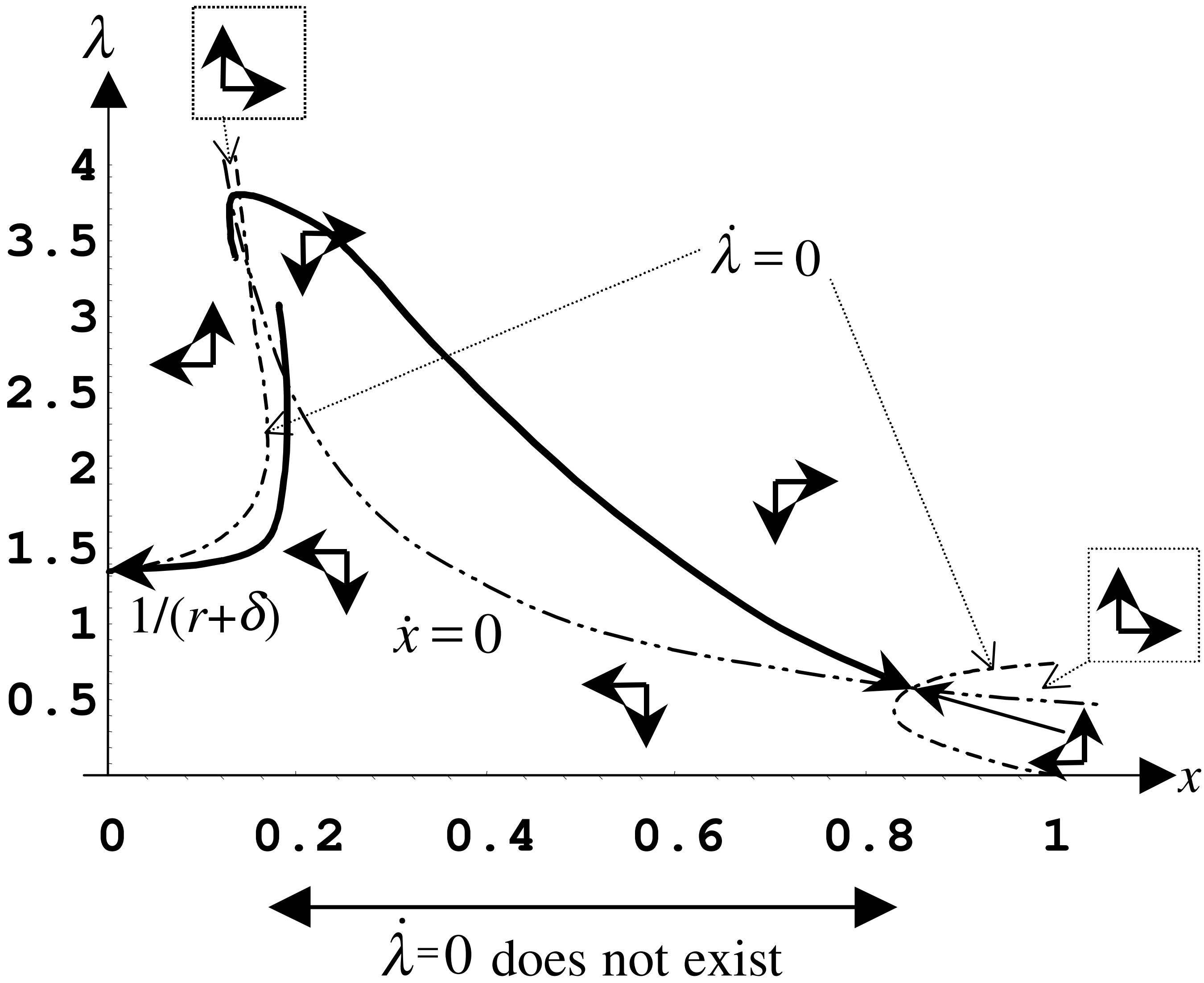

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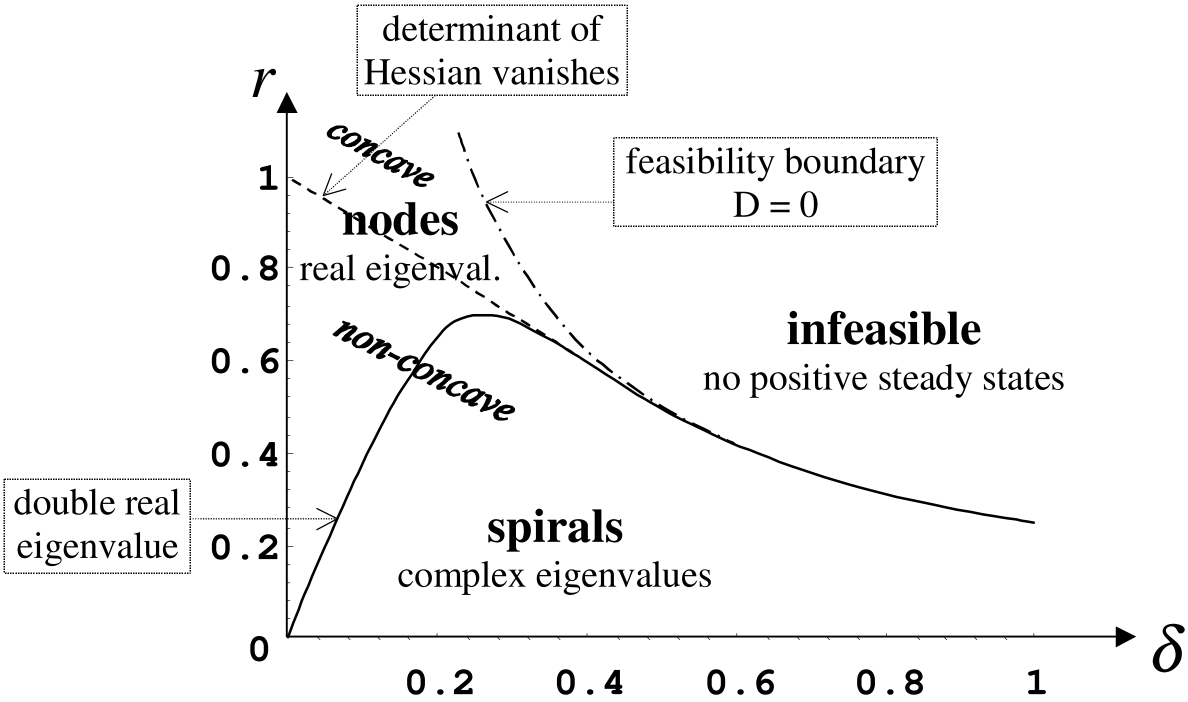

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