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Editorial

Any investment problem is inherently dynamic since the immediate outflow of cash to pay for the investment good has to be evaluated against the future net inflows of cash that the investment is expected to yield. The decision to extend a firm's labour force must depend on long-term predictions of the firm's performance, since it is not possible to get rid of workers without any costs in case future profits turn out to be worse than expected. Fixing the level of an environmental tax rate should be based on long-term economic growth predictions and the forecasts with respect to environmental issues like the greenhouse effect and the ozone layer. While building up a portfolio of stocks and bonds an investor is very interested in future economic developments.

This enumeration is by far not complete, but the point we want to make here is that a considerable part of economic science has a dynamic orientation. It is clear that understanding the dynamic aspects of an economic problem leads to better economic decisions. Therefore, it is important to establish a theoretical framework that could serve as a guidance for dynamic economic behaviour in practice. To contribute to this aim, dynamic mathematical techniques like control theory are applied to develop and analyse dynamic economic models.

The modelling and optimization of economic problems began in the 1970s, initiated by Arrow and his coworkers. Since then an ever increasing number of books, articles and surveys have appeared devoted to the application of optimal control theory and the theory of differential games to different areas of economics. From these contributions it is learnt that for a large number of highly relevant economic problems formal dynamic analysis could result in fruitful theoretical insights and important policy implications, but it also has become clear that advanced mathematical tools are required to obtain relevant results in realistic economic models. This *Special Issue on Economic Applications of Optimal Control* provides a state of the art collection of recent developments in economic applications of optimal control theory. The following survey of papers in this volume shows that applications evolve from topics in environmental economics to macroeconomics, while the mathematical tools range from optimal control models with mixed elements of both continuous and discrete-time problems, via distributed optimal control, optimal control with history-dependent equilibria, optimal control with stable and unstable steady states, and differential games, to decomposition of stochastic dynamic systems.

The contribution 'Optimal sequence of landfills in solid waste management' by Andre and Cerda deals with the design of an optimal sequence of landfills, as regarding their capacity and lifetime. Once a landfill is full it can be replaced at some cost, by constructing a new one. The new landfill will also be depleted and so on. Modelling this problem results into an optimal control model that involves splitting a time horizon of planning into subintervals the length of which has to be decided. It turns out that the optimal capacity of a certain landfill is determined such that marginal cost, consisting of the cost of building and management, equals marginal gain, which comes from all the discounted cost saving attached to future landfills. It is important to note that the modelling structure used here to solve the landfill problem can be applied to other economic problems as well. Think for instance of purchasing a personal computer:

purchasing a state-of-the-art computer implies a larger cost but is likely to have a longer lifetime, while a cheaper computer will become obsolete sooner.

In distributed optimal control the state and control variables are not a function of only time, but could also depend on, e.g. age or space. Due to its mathematical complexity, until now applications of this technique to economic problems are scarce, but it is important to notice that distributed optimal control has a lot to offer to economics because it allows for introducing heterogeneity in the model formulation. For instance, technological progress can be easily included into an optimal control model by distinguishing between different vintages of capital goods. Also, in problems of unemployment the population can be subdivided with respect to age: it seems to be more fruitful to spend money on retraining a younger person than on someone who is almost retired. Indeed, due to the increased possibilities to solve models numerically, analysing economic problems by distributed optimal control promises to become a fruitful research area. In their paper 'Using distributed optimal control in economics: a numerical approach based on the finite element method', Calvo and Goetz apply distributed optimal control to perform a numerical analysis of the optimal logging regime of a privately owned non-homogeneous forest. An important feature of this problem is that timber prices increase with the age of the tree. Therefore, except of time, it makes sense that the control and state variables are also a function of age.

A topical methodological area of research these days concerns Skiba points (sometimes also called DNS-point because of the crucial contributions by Dechert, Nishimura, and Skiba). A Skiba point is an initial state at which decision makers are indifferent between solutions converging to different long-run equilibria. These points are of interest since they separate very different long-run behaviour. The contribution 'Dynamic optimization and Skiba sets in economic examples', by Beyn, Pampel and Semmler provides a systematic numerical method for calculating locally optimal solutions and Skiba sets. Their method is applied to an optimal investment model where a Skiba point occurs, and to a model of optimal exploitation of renewable resources, where periodic solutions occur, but a Skiba point could not be detected. In the optimal investment model it turns out that the initial capital stock should exceed the capital stock level corresponding to the Skiba point in order to converge to a long run equilibrium with positive capital stock value. Otherwise, it is optimal to get rid of the complete capital stock in finite time.

The paper 'Local stability in endogenous growth models' by Pilar Martinez-Garcia investigates the local stability of balanced paths in endogenous growth models. The endogenous growth theory recognizes that knowledge can raise the return on investment. Whereas in the neoclassical theory technological progress 'just happens', in the endogenous growth theory knowledge is a factor of production which, like capital, has to be paid for by forgoing current consumption. In this way, technological progress is endogenized and it has been shown that this leads to the removal of decreasing returns to scale. This causes an endogenous sustained growth for the economy, and the aim of this paper is to contribute to the development of stability results suitable for sustained economic growth paths. A general instability theorem is established under the condition that transversality conditions are satisfied.

In the paper 'Solving dynamic macroeconomic policy games using the algorithm OPTGAME 1.0' by Hager, Neck and Behrens an algorithm, called OPTGAME 1.0, is presented that is able to determine equilibrium solutions of dynamic games in discrete time in case the objective functions are quadratic and the dynamic system is non-linear. OPTGAME can deliver open-loop and feedback Nash equilibrium solutions, open-loop and feedback Stackelberg equilibrium solutions and Pareto-optimal solutions. It is applied to a small macroeconometric model for Austria. It turns out that the time paths for the different equilibria are very similar.

In their contribution 'Near optimization of stochastic dynamic systems by decomposition and aggregation', Sethi and Zhang aim at specifying conditions that would justify the use of decomposition and aggregation in reducing a large stochastic optimal control problem to simpler problems. Solving these simpler reduced problems leads to nearly optimal controls for the original problem. This approach holds a significant promise in dealing with large dynamic stochastic macroeconomic problems. Consider, for instance, subsidizing a basic industry. In such a case (i) the relevant variables in the economy must be classified into a number of groups, (ii) the interaction within each group must be studied while ignoring interactions among groups, and then (iii) aggregate indices representing groups must be defined and the interaction among these indices has to be studied.

The articles in this special issue of *Optimal Control Applications & Methods* are based on selected presentations at the 7th Viennese Workshop on Optimal Control, Dynamic Games and Non-linear Dynamics: Theory and Applications in Economics and OR/MS, which was held in May 2000. We would like to thank all the participants of this Workshop for their contributions and enthusiasm which made this event a large success. Also, we are very grateful to the anonymous referees which provided us with reports on papers submitted for this special issue. Special thanks go to Bion Pierson who was instrumental in enabling us to edit this issue of OCAM. We look forward to seeing the future developments in this fascinating field in future Viennese Workshops.

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