

Optimal Control and Dynamic Games:  
Applications in Finance, Management Science,  
and Economics

In honor of Suresh Sethi

Aix en Provence, France, 2-6 Juin 2005

# PROGRAM

## Thursday June 02

From 20:00      **Registration and buffet at Aquabella**

## Friday June 03

From 8:00 on      Registration and welcome coffee at the conference room

08:50              Christophe Deissenberg and Richard Hartl: Opening

09:00-09:30      Alain Bensoussan: Suresh Sethi - Works and personality

### **9:30-10:30      Session 1: Production, Maintenance, and Transportation I. Chair: Janice E. Carillo**

Eugene Khmelnitsky and Gonen Singer: A Stochastic Optimal Control Policy for a Manufacturing System on a Finite Time Horizon

Helmut Maurer, Jang-Ho Robert Kim, and Georg Vossen : On a State-Constrained Control Problem in Optimal Production and Maintenance

10:30-11:00      **Coffee break**

### **11:00-12:30      Session 2: Marketing. Chair: Ngo Van Long**

Richard Hartl and Peter Kort: Advertising Directed Towards Existing and New Customers

Konstantin Kogan and Avi Herbon: A dynamic game between a wholesaler and a retailer under a limited-time promotion

Charles Tapiero: Advertising and Advertising Claims Over Time

12:30-14:30      **Lunch**

### **14:30-16:00      Session 3: Economics and Finance I. Chair: Charles Tapiero**

Gila E. Fruchter: Dynamic Brand-Image-Based Production Location Decisions

Jacek B. Krawczyk: Numerical Solutions to Lump-Sum Pension Fund Problems That Can Yield Left-Skewed Fund Return Distributions

Gerhard Sorger: Differentiated Capital and the Distribution of Wealth

16:00-16:30      **Coffee break**

### **16:30-18:00      Session 4: Economics and Finance II. Chair: Peter Kort**

Mikulas Luptacik: Data Envelopment Analysis in a dynamic framework

Andreas Novak: Finding optimal tax rates with conflicting social and financial goals

Ellina Grigorieva and Eugenie Khailov: Chattering Optimal Control Arising in a Microeconomic Problem of Profit Maximization

**FREE EVENING**

## Saturday June 04

From 8:30 on Welcome coffee at the conference room

09:30-10:30 **Section 5: Production, Maintenance, and Transportation II. Chair: Ali Dogramaci**

Janice E. Carrillo: The Impact of Dynamic Demand and Dynamic Net Revenues on Firm Clockspeed

Dirk Helbing and Stefan Lämmer: Self-Organized Control of Irregular or Perturbed Network Traffic

10:30-11:00 **Coffee break**

11:00-12:30 **Section 6: Methodological Advances. Chair: Helmut Maurer**

Alain Bensoussan: T.B.A.

Dean Carlson and George Leitmann: The Direct Method for a Class of Infinite Horizon Dynamic Games

Jerzy A. Filar and Boda Kang: Time Consistent Dynamic Risk Measures

12:30-14:30 **Lunch**

14:30-16:30 **Session 7: Environment. Chair: Gerard Sorger**

Hassan Bencheikroun, Ngo Van Long, and Seiichi Katayama: Capital Resource Substitution, Overshooting, and Sustainable Development

Masatoshi Fujisaki, Seiichi Katayama, and Hiroshi Ohta: Accumulation with Random Jump

Uri Shani, Yacov Tsur, and Amos Zemel: Characterizing Dynamic Irrigation Policies via Green's Theorem

Ngo Van Long: Transboundary Pollution Game with Asymmetric Payoff and Preferences

16:30-17:00 **Coffee break**

17:00-18:00 **Session 8: Economics and Finance III. Chair: George Leitmann**

Florian Wagener: Bifurcations in economic dynamic optimisation

Herbert Dawid and Christophe Deissenberg: Should we trust governmental announcements? A dynamic analysis

19:00 sharp Departure for the banquet at *Chateau de Meyrargues*. A bus will be waiting at the parking of Aquabella. Those who miss the bus will have to take a taxi (about 50 Euros). Any taxi driver will know the address. The phone number of Chateau de Meyragues is 04 42 63 49 90.

## Abstracts, in order of presentation

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Alain Bensoussan  
University of Texas at Dallas

*Suresh Sethi: Works and personality*

Abstract not available

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Eugene Khmelnitsky and Gonen Singer, Tel-Aviv University, Israel

*A stochastic optimal control policy for a manufacturing system on a finite time horizon*

We consider a problem of optimal production control of a single reliable machine. Demand is described as a discrete-time stochastic process. The objective is to minimize linear inventory/backlog costs over a finite time horizon. Using the necessary conditions of optimality, which are expressed in terms of co-state dynamics, we develop an optimal control policy. The policy is parameterized and its parameters are calculated from a computational procedure. Numerical examples show the convergence or divergence of the policy when the expected demand is greater or smaller than the production capacity. A non-stationary case is also presented

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Helmut Maurer, Jang-Ho Robert Kim, Georg Vossen, University of Muenster, Germany

*On a State-Constrained Control Problem in Optimal Production and Maintenance*

We consider a control problem introduced by Cho which "incorporates a dynamic maintenance problem into a production control model". For a quadratic production cost function we present a detailed numerical study of optimal control policies for different final times. The maintenance control is either composed by bang-bang and singular arcs or is purely bang-bang. In the case of a linear production cost, we show that both production and maintenance control are purely bang-bang. A recently developed second order sufficiency test is applied to prove optimality of the computed controls. This test enables us to calculate sensitivity derivatives of switching times with respect to perturbation parameters in the system. Furthermore, numerical results are presented in the case where a state constraint on the number of good items is added to the control problem.

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Richard Hartl, University of Vienna, Austria and Peter Kort, University of Tilburg, The Netherlands

*Advertising Directed Towards Existing and New Customers*

This paper considers a specific marketing problem based on a model by Gould (1970). The extension is that we have two kinds of advertising directed towards new customers and existing customers, respectively. We found that history dependent behavior occurs: if initial goodwill is small then it does not pay to spend a lot of money on advertising towards existing customers. Consequently convergence to a saddle point with low goodwill prevails where there is only advertising with the aim to attract new customers. On the other hand, for larger initial goodwill, eventually a steady state with a high goodwill level is reached where both types of advertising are used.

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Konstantin Kogan and Avi Herbon, Department of Interdisciplinary studies, Bar-Ilan University, Israel

*A dynamic game between a wholesaler and a retailer under a limited-time promotion*

We consider a two-echelon supply chain with a supplier and a retailer which face stochastic customer demands. The supplier is a leader who chooses its wholesale price. In response, the retailer orders products and sets up its price which affects customer demands. The goal of both players is to maximize their profit. We find the Stackelberg equilibrium and show that it is unique not only when the chain is in a steady state but also when it is in a transient state induced by a promotion. There is a maximum length to the promotion, however, so that an equilibrium still exists. Moreover, if the customer sensitivity increases during a promotion, then the wholesale equilibrium price decreases, product orders increase and product prices drop. This effect, well observed in real life, does not, however, necessarily imply that the promotion is always beneficial.

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Charles Tapiero, ESSEC, France, and Polytechnic University of New York

*Advertising and Advertising Claims over Time*

Advertising budget allocation with a carryover effects over time is a problem that was treated extensively by economists. Additional developments were carried out by both Sethi who has also provided some outstanding review papers and my own. The model treated by Sethi were essentially defined in terms of optimal control problems using deterministic advertising models while my own were essentially sales response stochastic models with advertising budget determined by stochastic control problems. These problems continue to be of academic and practical interest. Issues relating to the "advertising message" such as truthful claims advertising directed to first time buyers has not attracted much attention however.

The purpose of this paper is to address issues relating to advertising and their messages by suggesting a stochastic advertising-repeat purchase model. In this model, advertising directed to first time buyers is essentially defined by two factors: the advertising budget and the advertising message (such as statement regarding the characteristics of a product, its lifetime etc.). Consumers experience in case they buy the product will define the advertising message "reliability", namely that the probability that advertised message are confirmed or not. Repeat purchasers, however, are influenced by two factors, on the one hand the advertising messages that are directed to experienced consumers and of course the effects of their own experience (where past advertising claims whether truthful, or not, interact with customers' personal experience). Advertising claims that underestimate products characteristics might be "reliable" but then they might not entice first time purchasers, while overly optimistic advertising messages might entice first time purchasers but be perceived as unreliable by repeat purchasers who might switch to other competing brands. In this sense, the decision to advertise is necessarily appended by the decision to "what to advertise", which may turn out to be far more important for a firm. This paper provides a theoretical approach to deal with this issue.

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Gila E. Fruchter, Graduate School of Business Administration, Bar-Ilan University, Ramat-Gan 52900, Israel

*Dynamic Brand-Image-Based Production Location Decisions*

In this paper, we study the dynamic production location decisions of a manufacturer of a certain branded product. Considering brand-image as a form of goodwill, we extend the well-known Nerlove-Arrow dynamic model by adding both country-image and price. Formulating an optimal control problem for a group of countries in which the cost of production is convexly increasing with country-image, we are able to develop optimal decision rules for a manufacturer regarding the location of production and pricing over time. The resulted optimal policy has a very interesting pattern. Assuming that the demand rises by more than the corresponding change in brand-image, then, if brand image is increasing toward a stationary value level, the optimal policy should be to initially locate production in countries with high image and set a high price that signals high quality. Later, the production should gradually shift to countries with lower production costs and lower image and the price lowered until the stationary value level is reached. For brand-images beyond the stationary value level, the location of production should start in a country with low costs and country image while setting prices that signal

relatively low quality. Over time, production should be shifted to countries with gradually higher costs and images while setting higher prices until the brand-image approaches the level of stationary value.

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Jacek B. Krawczyk, KIER, Kyoto University; on leave from: Victoria University of Wellington, New Zealand

*Numerical Solutions to Lump-Sum Pension Fund Problems That Can Yield Left-Skewed Fund Return Distributions*

The paper is about pension fund problems where an agent pays an amount  $x_0$  to the fund manager and is repaid, after time  $T$ , a lump sum  $x(T)$ . Such problems admit an analytical solution for specific, rather unrealistic mathematical formulations. Several practical pension fund problems are converted in the paper into Markov decision chains solvable through approximations. In particular, a couple of problems with a non-differentiable asymmetric (with respect to risk) utility function are solved, for which left-skewed fund-return distributions are reported. Such distributions ascribe more probability to higher payoffs than the right-skewed ones that are common among analytical solutions.

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Gerhard Sorger, Department of Economics, University of Vienna

*Differentiated Capital and the Distribution of Wealth*

We present a one-sector growth model with finitely many households who differ from each other with respect to their endowments, their preferences, and the type of capital supplied to firms. There is monopolistic competition on the capital market and perfect competition on all other markets. We show that there exists a unique stationary equilibrium and that all households have strictly positive wealth in this equilibrium. We study how the stationary equilibrium depends on the time-preference rates of the households and on the elasticity of substitution between different types of capital. We also analyze the stability of the stationary equilibrium.

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Mikuláš Luptáčik, Vienna University of Economics and Business Administration, Austria

*Data envelopment analysis in a dynamic framework*

Data envelopment analysis (DEA) is well developed and widely used approach for measuring of efficiency. Based on the pioneering work of Farrel (1967), production efficiency has been measured as the distance between an observation and an efficiency frontier estimated in a way consistent with the underlining economic theory of optimizing behaviour. Using such an efficiency frontier as a benchmark, one can naturally define productive inefficiency. However, except for a few studies in the DEA literature, most previous work was dealing within a static framework and failed to model the intertemporal behaviour. The first steps towards the development of dynamic DEA have been taken by Sengupta (1995) and Färe-Grosskopf (1996). In the more recent papers Nemoto and Goto (1999), (2003) extended DEA to a dynamic framework by treating quasi-fixed inputs at the end of the period as if they were outputs in that period. The role of quasi-fixed inputs is 2-fold: they are put into today's production as "inputs" but are also treated as yesterday's "outputs", which introduces a dynamic interdependency across periods. This implies that a firm cannot hold more quasi-fixed inputs without giving up a certain amount of products. Furthermore, it also implies that the more quasi-fixed inputs are maintained, the more goods or services will be produced in the next period. The augmented DEA model can be formulated as linear programming problem from which the optimal condition for the adjustment path of quasi-fixed inputs are explicitly derived. This optimality condition is the Hamilton-Jacobi-Bellman equation in dynamic programming problem of a firm's optimizing behavior. Based on this equation a measure of dynamic inefficiency is obtained. An application to Japanese electric utilities over the 1981-1995 period by Nemoto-Goto (2003) delivered interesting results and proved the usefulness of the procedure.

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Andreas Novak, University of Vienna, Austria

*Finding optimal tax rates with conflicting social and financial goals*

Politicians and pressure groups often try to attain different goals with taxes. On one hand taxes are necessary to generate funds for public projects, on the other hand some taxes are used in order to change the behavior of those who have to bear them. Examples for that are taxes on certain drugs like tobacco and alcohol.

In an analytic static optimization model we determine the optimal tax rate when two conflicting goals like financial and social aims shall be attained to a certain degree. After deriving optimality conditions we proceed with a comparative static analysis.

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Ellina Grigorieva\* (Texas Woman's University, USA) and Eugenie Khailov (Moscow State University, Russia)

*Chattering Optimal Control Arising in a Microeconomic Problem of Profit Maximization*

The process of production, storage, and sales of a perishable consumer good is described by the following nonlinear system of three differential equations, and controlled by the rate of production.

$$\begin{cases} \dot{x}_1(t) = -n_p(Y - x_2(t))x_1(t) + u(t), & t \in [0, T] \\ \dot{x}_2(t) = n_p(Y - x_2(t))x_1(t) - k_1x_2(t) \\ \dot{x}_3(t) = pn_p(Y - x_2(t))x_1(t) - k_2x_1(t) - u(t) \\ x_1(0) = x_1^0 > 0, x_2(0) = x_2^0 > 0, x_3(0) = x_3^0 > 0 \end{cases}$$

The objective of this work is to find such optimal production strategy that maximizes firm's cumulative profit over certain time interval. Parameters of the model, for which singular control can be optimal, are defined. It is analytically proven that so-called "chattering control" appears as a link between optimal regular and optimal singular arcs. The time intervals on which chattering control occurs are found. Numerical results are obtained with the use of a computer program written in DELPHI. Possible economic applications are discussed.

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Janice E. Carrillo, Warrington College of Business, University of Florida

*The Impact of Dynamic Demand and Net Revenues on Firm Clock speed*

This paper yields analytic insights concerning the optimal pacing of new product development activities at the firm level. Using a simple analytic model, an optimal firm "clock speed" is derived. The derived clock speed is mostly dependent on marketing, technology, and operations related factors such as (i) average demand forecasts, (ii) dynamic profits earned over time, (iii) cannibalization of older products, and (iv) technology and/or production constraints limiting the pace of new product development.

Two different cases exist and may be appropriate for different types of firms. In the first type, the speed of new product introduction is not dictated by the organizational capabilities. In this type of industry, the optimal clock speed which maximizes profits for the firm actually lags the maximal pace that the firm could achieve. In contrast, the second case depicts the situation where the speed of new product introduction is dictated by the firm's organizational capabilities. In this case, an incentive exists for firms to invest in enhancing its production, design, and/or supply related capabilities to increase the frequency of new product introductions.

A key factor influencing firm clock speed is the anticipated shape of the demand/sales curve for each generation of a new product. Analytic results show that when demand curves are relatively flat, there is little incentive for the firm to introduce multiple generations of new products, particularly if development costs are formidable. In contrast, when demand curves are declining and development costs are low, the firm should introduce new products at the maximal pace possible with its current organizational capabilities. Finally, when the demand curves follow a traditional growth and decline pattern typically associated with the classical diffusion process, the firm optimally introduces each new generation of products after the old product has reached the declining phase of the product life cycle.

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Dirk Helbing and Stefan Lämmer, Dresden University of Technology

*Self-Organized Adaptive Signal Control in a Fluid-Dynamic Traffic Flow Model of Urban Queuing Networks*

We present a fluid-dynamic model for the simulation of urban traffic networks with street sections of different lengths and capacities. The model allows one to efficiently simulate the transitions between free and congested traffic based on an integrated Lighthill-Whitham model. On top of this, we observe non-linear dynamic patterns which are produced by the respective network topology. Synchronization is only one interesting example and implies the emergence of green waves. In this connection, we will discuss adaptive strategies of traffic light control which can considerably improve throughputs and travel times, using self-organization principles based on local interactions between vehicles and traffic lights. Similar adaptive control principles can be applied to other queuing networks such as production systems.

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Alain Bensoussan University of Texas at Dallas

*T.B.A.*

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Dean A. Carlson, Mathematical Reviews and George Leitmann, University of California at Berkeley

*The Direct Method for a Class of Infinite Horizon Dynamic Games*

In this paper we present an extension of a direct solution method, originally due to Leitmann for single-player games on a finite time interval, to a class of infinite horizon N-player games in which the state equation is affine in the strategies of the players. Our method, based on a coordinate transformation method, gives sufficient conditions for an open-loop Nash equilibrium. An example is presented to illustrate the utility of our results.

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Jerzy A. Filar and Boda Kang, University of South Australia

*Time Consistent Dynamic Risk Measures*

Abstract not available

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Hassan Benchenkroun, McGill University, Ngo Van Long, McGill University, Seiichi Katayama, Kobe University

*Capital Resource Substitution, Overshooting, and Sustainable Development*

In this paper, we study the optimal path for an economy that produces an output using a stock of capital and a resource input extracted from a stock of renewable natural resource. We retain the Solow-Dasgupta-Heal assumption that capital and resource are substitutable inputs in the production of the final good, but our model differs from theirs because the resource stock is renewable. We wish to find the optimal growth path of the economy under the utilitarian criterion. We show that there exists a unique steady state with positive consumption. We ask the following questions: (i) Can it be optimal to get to the steady state in finite time under the assumption that the utility function is strictly concave? (ii) Can finite-time approach paths to the steady state be smooth, in the sense that there are no jumps in the control variables? (iii) Are there non-smooth paths to the steady state?

We show that starting from low levels of capital stock and resource stock, the optimal policy consists of three phases. In phase I, the planner builds up the stock of man-made capital above its steady state level, while the resource stock is kept below its steady state level. In phase II, the capital stock declines steadily, while the resource stock continues to grow, until the steady state is reached. In phase III, the economy stays at the steady state. Thus, our model exhibits the "overshooting" property.

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Masatoshi Fujisaki, Seiichi Katayama, and Hiroshi Ohta, Kobe University, Japan

*Common Property Resource and Private Capital Accumulation with Random Jump*

Long and Katayama (2002) presented a model of exploitation of a common property resource, when agents can also invest in private and productive capital. They considered the case where the resource extracted from a common pool is non-renewable. In this paper, we try to extend their result to the case where the common pool is under uncertainty in the sense that it could have a sudden increase or decrease in the process of extraction and moreover we shall calculate the exhaustion probability of the resource.

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Uri Shani, the Hebrew University of Jerusalem, Yacov Tsur, The Hebrew University of Jerusalem and Amos Zemel, Ben Gurion University of the Negev

*Characterizing Dynamic Irrigation Policies via Green's Theorem*

We derive irrigation management schemes accounting for the dynamic response of biomass yield to salinity and soil moisture as well as for the cost of irrigation water. The turnpike structure of the optimal irrigation policy is characterized using Green's theorem. The analysis applies to systems of arbitrary end conditions. A numerical application of the turnpike solution to sunflower growth under arid conditions reveals that by selecting the proper mix of fresh and saline water for irrigation, significant savings on the use of freshwater can be achieved with negligible loss of income.

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Ngo Van Long, McGill University

*Transboundary Pollution Game with Asymmetric Payoff and Preferences*

We study a model of transboundary pollution where countries have asymmetric payoff and preferences. Countries have different technological coefficients of emissions and abatements. A Markov perfect equilibrium is found for each set of parameter values. The rich country can induce the poor country to reduce emissions by offering an income transfer scheme. Under our assumption, the fraction of income transferred per period turns out to be a constant.

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Florian Wagener, University of Amsterdam

*Bifurcations in economic dynamic optimization*

Abstract not available

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Herbert Dawid, University of Bielefeld and Christophe Deissenberg, Université de la Méditerranée and Greqam

*Should we trust governmental announcements? A dynamic analysis*

We consider a version of the seminal Kydland-Prescott model where, in each period, some private agents believe the policy announcements made by the government. The other agents follow a standard optimizing strategy. The fraction of agents who believe the government changes over time according to a word-of-mouth learning process. We show that the initial number of believers and the speed of learning can have drastic consequences for the policy followed and the losses experienced by the different agents. In particular, the utility of the private sector may jump upwards if the initial number of believers exceeds a given threshold.

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## List of participants

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